Integration of Multimodal Transportation Services

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Integration of Multimodal Transportation Services

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16. Abstract

Flexible route paratransit services may complement as well as compete with conventional public transportation services (that have fixed routes and schedules). Flexible routes are especially suitable for service areas or time periods with low demand densities (and especially rural areas) and may be used to concentrate the low demand for conventional bus and rail services. Excess drivers and vehicles from conventional services can be leveraged to provide higher quality door-to-door services during off-peak periods. In the proposed project, practical methods will be developed for planning and operating integrated multi-modal public transportation services. In particular, these methods focus on (a) formulating demand relations for integrated multi-modal transportation services, (b) improving the efficiency of algorithms for managing ridesharing and taxi services, (c) improving the coordination of transfers among vehicles from various routes and modes, and (d) exploring the potential benefits of managing demand through service options, pricing and other incentives.

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STUDY BACKGROUND

Conventional services (with fixed routes and schedules), ridesharing services such as Dial-A-Ride (DAR), and taxis offer a spectrum of service types with various combinations of service quality, capacity and cost. Normally, conventional services must have sufficient vehicles and drivers for peak period requirements, but these are largely wasted at most other times. By switching vehicles and drivers among different types of services as demand changes cyclically over time, we can offer more responsive door-to-door services during off peak periods and transform the excess capacity of conventional services into a range of DAR services with lower capacity but higher quality. In turn, the efficiency of DAR services can be enhanced by outsourcing the trips that are unsuitable for ridesharing (mainly those very long and away from densely populated areas) to taxis. By thus providing services of higher quality during off peak periods, some users can be diverted from peak periods, thereby reducing total resource requirements and congestion effects. Such beneficial demand management as well as other efficiencies may be further promoted with reservations, incentives and pricing options. Further synergies of integration may be achieved (1) by using DAR services and taxis to concentrate passengers for conventional services rather than just compete with or substitute for them and by (2) coordinating the arrivals of vehicles at transfer stations through efficient routing, scheduling, slack time in schedules, real-time dispatching decisions at transfer stations, and traffic signal control.

This project develops practical tools for planning and operating integrated multimodal public transportation systems that are implemented in realistic urban networks. It also develops mode choice models to better predict how public transit users will select between traditional fixed transit options and the newly emerging flexible types of public transit services.

This report presents three research efforts are either published in an academic journals or being prepared for publication. The remainder of this report provides a short description of each of these studies. The latest draft of each of these studies are then included in the Appendices of this report.

Paper 1: Integration of conventional and flexible bus services with timed transfers (published in *Transportation Research Part B*)

This paper focuses on the problems of optimizing integrated public transportation systems while also coordinating passenger transfers among vehicles at one major transfer terminal, despite randomness in vehicle travels times and consequent variability in their arrival times at the transfer terminals. This is accomplished by optimizing probabilistic slack times for each route, in addition to optimizing other key decision variables such as headways, vehicle sizes and service zone sizes. The slack times constitute “reserve” or “safety” factors which significantly reduce the probability of missed connections resulting from variability in bus arrival times. The transfer delays and other costs related to probabilistic variations in demand and travel times can be further reduced by optimizing dispatching decisions, as shown in previous studies by our team (Lee & Schonfeld 1994, Ting & Schonfeld 2007).

This paper is presented in Appendix A.

Paper 2: Welfare maximization for bus transit systems with timed transfers and financial constraints (published in *Journal of Advanced Transportation*)

The main contributions of this paper is in combining the following problem aspects: integration of conventional and flexible services, welfare maximization with demand elasticity, multiple regions, and financial constraints. Specifically, elastic demand, multiple regions and financial constraints are three important characteristics of a public transportation system, corresponding to human behavior, network topology and finance. The joint consideration of the above four aspects significantly extends the realism and value of the analysis. Furthermore, the additional efficiencies obtainable by combining service types and coordinating transfers among routes can be substantial.

The problem is modeled with an objective of optimizing service type, headways, fares, and service zone sizes for each region in a multiregional bus transit system. The solution is sought with a hybrid algorithm in which the problem is subdivided into two subproblems, and a genetic algorithm with bounded integer variables is implemented to solve one of the subproblems. A numerical example and analyses of sensitivity to various input parameters are then used to demonstrate the method's capabilities.

Sensitivity analyses yield the following findings. The conventional services are increasingly preferable at high demand densities and low values of the access time elasticity parameter. Flexible services are preferable at low demand densities and high access time elasticities. No significant effects of maximum allowable subsidy on the optimal service types are observed. To precisely determine the most suitable service types for various regions and periods, detailed input data can be used to optimize each alternative service type for each region under various conditions. For all tested input parameters, the optimized headways in the coordinated system for all three regions are the same, so that all bus routes are fully synchronized to eliminate the transfer delays, assuming adherence to the preset schedules. Therefore, for coordinated systems, if a full search of possible headway combinations is considered too costly, the headway search can be focused on common headways or integer-ratio headway. Compared to uncoordinated operations, coordinated operations were found to enhance welfare by 5%–20%, depending on circumstances. Coordination increases welfare at high demand and low subsidy. However, the relative benefits of coordination are higher at low demand.

The analyses show the interrelations among various parameters and the system performance measures for relatively complex transit systems with elastic demand, multiple service types, timed transfers, and financial constraints. Such results can support the efficient planning and design of realistic transit systems.

This paper is presented in Appendix B.

Paper 3: Development of a mode choice model for general purpose flexible route transit systems (being prepared for submission)

This paper develops a mode choice model that can be used to unveil how transit users select between competing transit options. Specifically, the model choice model considers traditional fixed-route transit systems, flexible-route systems in which vehicles are shared but routes are flexible to prevailing demands and individual transit systems (e.g., Uber or Lyft) that provide door-to-door and demand responsive service. A stated preference survey was performed in which survey participants were provided a set of scenarios and asked to select the most attractive transit option of the three previously mentioned. Each scenario was presented using the following attributes: walking time required, waiting time (including variability), in-vehicle travel time (including variability), monetary cost and availability of GPS tracking services. Various statistical modeling frameworks were considered and applied to these survey data to describe the mode choice decision-making process. The results revealed that some individuals always select the same mode, regardless of the parameters. However, costs, expected in-vehicle waiting time, expected waiting time and walking time were found to be statistically significant predictors of the type of transit option selected.

This paper is presented in Appendix C.

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APPENDIX A

Integration of Conventional and Flexible Bus Services with Timed Transfers

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INTEGRATION OF CONVENTIONAL AND FLEXIBLE BUS SERVICES WITH TIMED TRANFERS

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ABSTRACT

Conventional bus services, which have fixed routes and fixed service schedules, and flexible bus services, which provide doorstep services, have different advantages and disadvantages, with conventional services being generally preferable at high demand densities and flexible services being preferable at low densities. By efficiently integrating conventional and flexible services and thus matching service type to various regions, the total cost of transit services may be significantly reduced, especially in regions with substantial demand variations over time and space. Additionally, transit passengers must often transfer among routes because it is prohibitively expensive to provide direct routes for among all origin-destination pairs in large networks. Coordinating vehicle arrivals at transfer terminals can greatly reduce the transfer times of passengers. In this paper, probabilistic optimization models, which are proposed to deal with stochastic variability in travel times and wait times, are formulated for integrating and coordinating bus transit services for one terminal and multiple local regions. Solutions for decision variables, which include the selected service type for particular regions, the vehicle size, the number of zones, headways, fleet, and slack times, are found here with analytic optimization or numerical methods. The proposed models generate either common headway or integer-ratio headway solutions for timed transfer coordination based on the given demand. A genetic algorithm is proposed as a solution method and tested with numerical examples.

Key Words: Integration, Schedule Coordination, Conventional Bus, Flexible Bus, Genetic Algorithm, Analytic Optimization

INTRODUCTION

In urban bus transit networks, passengers must commonly transfer among routes because it is prohibitively expensive to provide direct routes among all origins and destinations with conventional bus services, which have fixed routes and fixed schedules. Since transfers are important in public transportation services, it may be beneficial to coordinate (i.e. synchronize) bus arrivals at transfer stations (terminals) so that such "timed transfers" help minimize wait times and passengers can reliably catch their next bus. Because bus travel times and dwell times are usually stochastic, a probabilistic optimization model is needed for determining buffer (slack) times that help provide reliable connections among buses.

Different bus services have different characteristics. Conventional services can carry numerous passengers at low average costs per round trip, but their average costs are very high at low demand densities. Flexible services, which are defined here as demand-responsive services having flexible routes with different pick-up and drop-off stops on each successive "tour", typically carry fewer passengers than conventional services. Flexible services may offer door-to-door service (subject to the physical accessibility of those doors) and are generally preferable for low demand densities. Thus, by effectively integrating conventional and flexible services the overall system cost may be significantly reduced in comparison with pure conventional or flexible services (Chang and Schonfeld 1991; Kim and Schonfeld 2012; Kim and Schonfeld 2013), especially when the demand density varies considerably over time and space.

In this paper we explore the potential reductions in system costs that are achievable by combining coordinated timed transfers with the integration of conventional and flexible services. The resulting system costs are compared with those of uncoordinated conventional and flexible services. To do this we develop a probabilistic optimization model for timed transfers with integration of conventional and flexible bus services. Uncoordinated and coordinated bus operation models are also formulated and compared through numerical analyses. The optimization model for uncoordinated operations finds solutions for the service type selection, vehicle size, headway, fleet size, and number of zones for both conventional and flexible services. In order to efficiently coordinate vehicles for timed transfers, common headways or integer-ratio headways are found numerically. For integer ratio headways, a round travel time is an integer multiple of a base cycle.

LITERATURE REVIEW

Kyte et al (1982) present a timed-transfer system in Portland, Oregon, which provides services since 1979, including its planning history, implementation, and evaluation. This system provides timed transfers to the suburban areas in which demand densities are low, and provides grid-type bus services for higher demand regions. The authors also discuss the performances and results of the implemented system. They use two indicators, which are a successful meet and a successful connection, to analyze the transfer reliabilities. A successful meet is defined as all buses arriving as scheduled at a given time, and a successful connection is a direct transfer connection that results from two routes arriving as scheduled. The authors point out that weekday ridership increased by 40 percent after one year of operation, and local trips using this system increased dramatically. However, the 40% increase of ridership results not only from the timed transfer system, but also from new route designs. Bakker et al (1988) similarly study a multi-centered time transfer system in Austin, Texas, and confirm that such timed transfers system are particularly applicable for low-density cities.

Abkowitz et al (1987) study timed transfers between two routes. They compare four policy cases, namely unscheduled, scheduled transfer without vehicle waiting, scheduled transfers with the lower frequency bus being held until the higher frequency vehicle arrives, and scheduled transfers when both buses are held until a transfer event occurs. In other words, they compare scheduled, waiting/holding, and double holding transfer strategies. They note that the effectiveness of timed transfers varies with route conditions. However, they find that the scheduled transfers are effective (over the unscheduled) when there is incompatibility between headways and the double holding strategy outperforms the other time transfer strategies when the headways on intersecting routes are compatible. They also note that slack time should be built into the schedule so that vehicle holding does not cause significant delays to passengers.

Domschke (1989) explores a schedule coordination problem with the objective of minimizing waiting times. He provides a mathematical programming formulation which is generally applicable to public mass transit networks such as those using subways, trains and/or buses. The formulation is a quadratic assignment problem. With four routes and five transfer stations in a very simple network, this paper uses heuristics and a branch and bound algorithm. The heuristics include a starting heuristic, which is based on rigid regret heuristic, and then a heuristic improvement procedure. Lastly, simulated annealing (SA) is applied to improve the solutions. For SA, the quality of the initial solution is important. The author finds that problems with more than 20 routes cannot be solved with exact solution methods.

Knoppers and Muller (1995) provide a theoretical note on transfers in public transportation. Their main objective is to minimize the passengers' transfer time. They find that when the frequency on the connecting lines increases, the benefit of transfer coordination decreases. Muller and Furth (2009) seek to reduce passenger waiting time through transfer scheduling and control. They provide a probabilistic optimization model, and discuss three transfer control types, namely departure punctuality control, attuned departure control, and delayed departure of connecting vehicles. They confirm that by increasing a buffer (slack) time, the probability of missing the connection decreases. However, a larger buffer increases the transfer time for riders who do not miss their connection. They also find that if the control policy allows a bus to be held to make a connection, the optimal schedule offset, which is the time between the arrival time of the feeder vehicle and the departure time of the connecting vehicle, decreases.

Shrivastava et al (2002) first discuss existing algorithms for solving nonlinear mathematical programming, because transit scheduling problems are often nonlinear. The existing algorithms are generally gradient-based, and require at least the first order derivatives of both objective and constraint functions with respect to the design variables. Gradient-based methods can identify a relative optimum closest to the initial guess of the optimum design. However, there is no guarantee of finding the global optimum if the design space is known to be non-convex. In such a case, exhaustive and random search techniques such as random walk or random walk with direction exploitation are quite useful. The main drawback with these methods is that to reach a good solution they often require thousands of function evaluations, even for the simplest functions. They also note that genetic algorithms (GAs) are based on exhaustive and random search techniques, and are robust for optimizing nonlinear and non-convex functions. Thus, they apply a genetic algorithm (GA) to schedule coordination problems. Their objective function includes waiting time, transfer time, and in-vehicle time for users, and vehicle operating cost for operators. They try to solve routing and scheduling simultaneously. The GA is designed with two substrings where one represents routes, and the other represents frequencies on those routes. By solving benchmark problems, they find that genetic algorithms provide better solutions than other heuristics. They also note that computational times are proportional to the population size. Cevallos and Zhao (2006) also use a GA to solve a transfer time optimization problem for a fixed route system. Their main focus is efficient computation time.

Lee and Schonfeld (1991) study optimal slack times for coordinating transfers between rail and bus routes at one terminal. The transfer cost function is formulated as a sum of scheduled delay cost, missed connection cost for bus to train transfer, and missed connection cost for train to bus transfer. The rail transit line is assumed to run on-time (without slack), while slack times for bus routes are optimized analytically. Bus arrivals are assumed to vary independently from train arrivals so that the joint probabilities of arrivals may be obtained by simply multiplying the probabilities obtained separately from the bus and train arrivals distributions. They show that analytic optimization, even with simplifying assumptions, is limited and difficult to solve for complex situations. Thus, they develop a numerical optimization method to find solutions efficiently.

Ting and Schonfeld (2005) extend Lee and Schonfeld (1991)'s study to bus service coordination among multiple transit routes in multiple hub networks. They compare uncoordinated and coordinated operations. For uncoordinated operation, their formulation minimizes the total system cost which is sum of operating cost, user waiting cost, and user transfer cost. Transfer cost in uncoordinated operation is simply assumed to be the product of the average transfer waiting time and the total number of transfer passengers. For the coordinated operation, the transfer cost consists of slack-time cost, missed connection cost, and dispatching delay cost. Common headway and integer-ratio headway cases are optimized with a heuristic algorithm. Their algorithms and numerical results determine when coordinated operations with integer-ratio headways are preferable over uncoordinated operation in terms of total cost. Simplifying assumptions of this work are that: 1) only one dispatching strategy is considered, which means vehicles do not wait for other vehicles that arrive behind schedule; 2) vehicle arrivals on a route are assumed to vary independently from those of other routes, so that the joint probabilities of arrivals may be obtained by simply multiplying the probabilities obtained separately from the two vehicle arrival distributions. A limitation of this work is that it does not ensure integer fleet size.

Chen and Schonfeld (2010) adapt the concept of bus transit coordination methods to freight transportation. They follow the main ideas of joint probabilities and transfer cost components from some previous transit studies (Lee and Schonfeld, 1991; Ting and Schonfeld, 2005). In this study, they propose two solution approaches, namely a genetic algorithm and sequential quadratic programming (SQP) to find good solutions for frequencies and slack times in intermodal transfers.

Chowdhury and Chien (2002) also study the coordination of transfers among rail and feeder bus routes. Their objective is to minimize total cost, including supplier and user costs, as in that other studies. They explore various degrees of coordination such as full coordination, partial coordination, and no coordination. They also follow the assumption of joint probabilities of independent vehicle arrivals, and assume that trains operate on-time. Recently, Chowdhury and Chien (2011) extend a previous study by jointly optimizing bus size, headway, and slack time for timed transfers. They optimize bus size by assuming maximum allowable bus headways instead of minimum cost headways. Therefore, their optimized bus size may be overestimated. Powell's algorithm (i.e., multi-variable numerical optimization) is used to solve this problem. Unfortunately, they do not present enough details on the methodology to clarify how decision variables are jointly optimized and how variables are constrained to be integer. Another limitation of this study is that although it optimizes vehicle size jointly with other decision variables, such as headways and slack times, the vehicle size is optimized for only one time period. Thus, optimizing vehicle size and required fleet size for daily demand or system-wide demand while finding headways and slack times for each time period offer an opportunity for improvement.

Chandra and Quadrifoglio (2012) develop an analytic model for estimating the cycle length of a demand responsive feeder for the maximum service quality, which is defined as the inverse of a weighted sum of waiting and riding time. They consider the demand responsive operations as a queuing problem. Their formulation does not have a line-haul distance between the terminal and service area. The analytic model for estimating the cycle length is designed for a vehicle within a rectangular service area. The waiting time and riding (in-vehicle) times are used in the objective function, but operating cost is not included. The vehicle size and fleet size are not optimized. The formulation is solved with uniform demand distributions and evaluated with simulation data. This study is of interest for improving flexible services, especially for replacing Stein's formula (1978), and for integration of conventional and flexible services. This study is limited to analyzing flexible services' cycle lengths, without seeking to integrateor coordinate conventional and flexible services.

Nourbakhsh and Ouyang (2012) propose an analytic model for a structured flexible transit system, along models for fixed transit and taxi services. They used user and operator costs for the system evaluation. They find that for low-to-moderate demand densities flexible transit systems are preferred to fixed transit or taxi services, as aslo noted by Kim and Schonfeld (2012 and 2013)

Jonge and Teunter (2013) consider walking as a transportation mode. They formulate intermodal public transportation network which allows passengers to alternate their rides with (short) walks. They argue that these walks might be worthwhile, especially when demand densities are high. The main purpose of this study is to analyze the trade-offs between the overall travel times versus the number of walks. The walking distances are Euclidean ones multiplied by 1.25, which is the circuitry factor. This paper assumes that time table and routes are given. This paper has no discussion of passengers' discomfort that is caused by the number of walks within their trips.

Ibarra-Rojas and Rios-Solis (2012) maximize passenger transfers while preventing bus bunching. The problem is formulated as linear integer problem and Multi-start iterated local search (MSLS) is proposed as a heuristic method. The proposed solution method is compared with a branch and bound algorithm. Their formulation is based on the assumption that the bus network is private, which can be realistic in some transit networks. This paper is limited to conventional services. It does not consider the effects bus holding policieson transfers, operating cost and some user costs such as in-vehicle and access costs.

Jara-Diaz et al, (2012) optimize the bus service frequencies, fleet, and vehicle size along with the line structures for fixed route bus services. The line structures are defined as single line, lines with transfers, exclusive lines, and shared lines. Their analytic model finds that the optimized line structure is governed by the directional demand and the cost components.

No previous study is found for coordinating transfers with integration of conventional and flexible bus services, although there are studies on timed transfers coordination for other modes. In evaluating coordination types, this paper explores uncoordinated (and hence independently optimized) headways and coordinated headways (i.e., common headways or integer-ratio headways), as explained below. The formulations developed here also consider multiple regions. The solution method proposed here also ensures integer vehicle size(s) as well as integer fleet size(s). Various analyses of sensitivity to important input values are also conducted.

BUS SERVICES AND ASSUMPTIONS

The delayed connection cost, missed connection delay cost, and slack time delay cost are considered in the transfer cost for the timed transfers formulation. Route travel times (i.e., bus arrivals at timed transfer station) are be assumed to be stochastic. To deal with stochastic bus arrivals, a probabilistic optimization problem is formulated and solved. For timed transfer analysis, it is assumed that vehicle arrivals at timed transfer terminals are probabilistically distributed and independent of each other.

The system analyzed here provides bus transit services between the main terminal and multiple local regions (shown in Figure 1). These local regions may fill up the space around the main terminal. For each region, either conventional or flexible bus service is provided, as shown in Figure 2. Detailed descriptions and assumptions for conventional and flexible bus services are provided below.

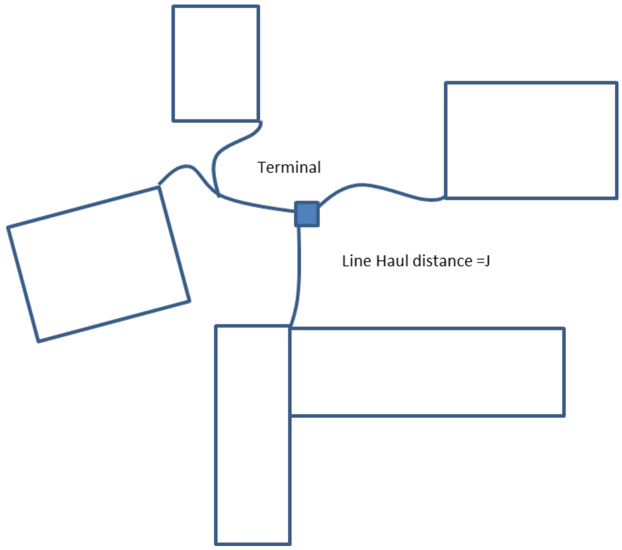


Figure 1 Transfer Terminal and Local Regions

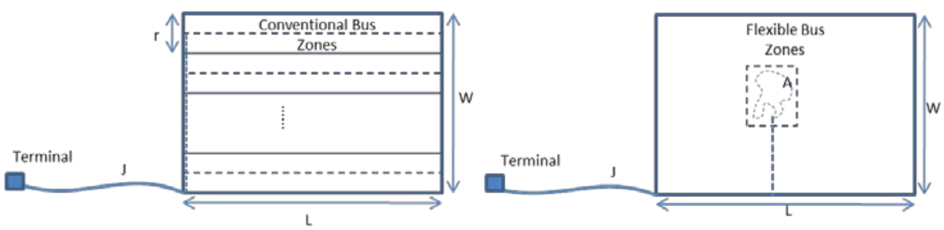


Figure 2 Conventional and Flexible Bus Services

Assumptions

Assumptions are adapted for consistency with other relevant studies (e.g., Kim and Schonfeld, 2012), and are modified to consider timed transfers in the terminal. Superscript k corresponds to regions, and subscripts c and f represent conventional and flexible bus services, respectively. Definitions, units, and default values of variables are presented in Table 1.

Table 1 Notation

Variable	Parameter	Definition	Baseline Value
	a	hourly fixed cost coefficient for operating bus ($/bus min)	0.833
A^k		service zone area(mile^2)= L^kW^k/N'	-
	b	hourly variable cost coefficient for bus operation ($/seat min)	0.0033
	$C_{tc,c}^{u,k}, C_{tc,f}^{u,k}$	uncoordinated total service cost over region k subscript: tc = total cost, c = conventional, f=flexible	-
	$C_{oc,c}^{k}, C_{oc,f}^{k}$	operating cost for region k subscript: oc = operating cost, c = conventional, f=flexible	-
	$C_{ic,c}^{k}, C_{ic,f}^{k}$	in-vehicle cost for region k subscript: ic = in-vehicle cost, c = conventional, f=flexible	-
	$C_{wc,c}^{k}, C_{wc,f}^{k}$	waiting cost for region k subscript: wc = waiting cost, c = conventional, f=flexible	-
	$C_{fc,c}^{u,k}, C_{fc,f}^{u,k}$	uncoordinated transfer for region k subscript: fc = transfer cost, c = conventional, f=flexible	-

	C_s C_i C_m C_d	costs for timed transfers subscript: s: slack time cost, i: inter-cycle waiting cost, m: missed connection cost, d: delayed connection cost	-
	C_{xc}^k	conventional service access cost for region k subscript: xc = access cost	-
	D_c^k	distance of one flexible bus tour in local region k (miles)	-
	D_f^k	equivalent line haul distance for flexible bus on region k $(=(L^k+W^k)/z+2J^k/y)$, (miles)	-
	D^k	equivalent average bus round trip distance for conventional bus on region k (= $2J^k/y+W^k/z+2L^k$),(miles)	-
	d	bus stop spacing (miles)	0.2
	f^k	directional demand split factor in region k	-
F_c^k, F_f^k		fleet size for region k (buses) subscript corresponds to (c = conventional, f=flexible)	-
	g_{jk}	the greatest common divisor of β_j and β_k	-
h_c , h_c^k		headway for conventional bus; for region k (min /bus)	-
h_f , h_f^k		headway for flexible bus; for region k (min /bus)	-
	$h_c^{k,max}, h_f^{k,max}$	maximum allowable headway for region k; subscript: c = conventional, f=flexible	-
	$h_c^{k,min}, h_f^{k,min}$	minimum cost headway for region k; subscript: c = conventional, f=flexible	-
	k	index (k: regions)	-
	J^k	line haul distance of region k (miles)	-
	l_c, l_f	load factor for conventional and flexible bus (passengers/seat)	1.0
	L^k, W^k	length and width of local region k (miles)	-
	M^k	equivalent average trip distance for region k $(=J^k/y_c+W^k/2z_c+L^k/2)$	-
	n	number of passengers in one flexible bus tour	-
N_c^k, N_f^k		number of zones in local region for conventional and flexible bus	-
	Q^k	round trip demand density in region k (trips/ min)	-
	Q_t^k	transfer demand density in region k (trips/ min)	-
	Q_{nt}^k	non-transfer demand density in region k (trips/min)	-
	Q_{jk}	transfer demand from region j to k (trips/min)	-
r^k		route spacing for conventional bus at region k (miles)	-
	R_c^k	round trip time of conventional bus for region k (min)	-
	R_f^k	round trip time of flexible bus for region k (min)	-
S_c^k, S_f^k		conventional and flexible bus sizes for region k (seats/bus)	{7,10,16,25,35,45}
s_k		slack time for region k (min)	-
	t	substitution variable for flexible service headways	-
	u	average number of passengers per stop for flexible bus	1
	V_c	local service speed for conventional bus (miles/ min)	0.5
	V_f	local service speed for flexible bus (miles/ min)	0.417
	V_x	average passenger access speed (mile/ min)	0.0417
	v_v, v_w, v_x, v_f	value of in-vehicle time, wait time, access and transfer time ($/passenger min)	0.167, 0.167, 0.25, 0.24
y		base cycle for integer ratio headway	-
	z	non-stop ratio = local non-stop speed/local speed	conventional bus = 1.8 flexible bus = 2.0
	z_{jk}	Transfer time from region j to region k	-
	$Ø$	constant in the flexible bus tour equation (Daganzo, 1984) for flexible bus	1.15
	σ	standard deviation of travel times in minute	0.5, 1, 0.3, 0.5
β^k		integer number for integer ratio headway in region k	-
	*	superscript indicating optimal value; subscript: c = conventional, f=flexible	-

All service regions, 1… k, are rectangular, with lengths L_k and widths W_k. These regions may have different line haul distances J_k (miles, in region k) connecting the main terminal and each region's nearest corner.

Assumptions for both conventional and flexible services

a) The demand is fixed with respect to service quality and price.

b) The demand is given and uniformly distributed over space within each region.

c) Within each local region k, the average speed (V_c for conventional bus, V_f for flexible bus) includes stopping times.

d) Passenger arrivals at each stop are random and uniformly distributed.

e) Layover times are not considered in the formulation

f) External costs are assumed to be negligible.

g) Transfer coordination is analyzed under the no-vehicle holding policy.

Assumptions for conventional services

a) The bus sizes (S_c for conventional services) that minimize the total cost for each region are selected from a discrete list of possible sizes.

b) The region k is divided into N_c^kN parallel zones with a width $r^k = W^k/N_c^k N$ for conventional bus, as shown in Figure 6-2. Local routes branch from the line haul route segment to run along the middle of each zone, at a route spacing $r^k = W^k/N_c^k N$.

c) Q^k trips/hour, entirely channeled to or from (or through) the single terminal, are uniformly distributed over the service area.

d) In each round trip, as shown in Figure 6-2, buses travel from the terminal a line haul distance J^k at non-stop speed zV_c to a corner of the local regions, then travel an average of $W^k/2$ miles at local non-stop speed zV_c from the corner to the assigned zone, then run a local regionof length L^k at local speed V_c along the central axis of the zone while stopping for passengers every *d* miles, and then reverse the above process in returning to the terminal.

Assumptions for flexible services

a) The bus sizes (S_f for flexible services) are jointly optimized with other decision variables for each region.

b) To simplify the flexible bus formulation, region k is divided into $N_f^k N'$ equal zones, each having an optimizable zone area $A^k = L^k W^k / N_f^k N'$. The zones should be "fairly compact and fairly convex" (Stein, 1978).

c) Buses travel from the terminal line haul distance J^k at non-stop speed zV_f and an average distance $(L^k+W^k)/2$ miles at local non-stop speed zV_f to the center of each zone. They collect (or distribute) passengers at their door steps through an efficiently routed tour of n stops and length D_c^k at local speed V_f . D_c^k is approximated according to Stein (1978), in which $D_c^k = \emptyset\sqrt{nA^k}$, and $\emptyset = 1.15$ for the rectilinear space assumed here (Daganzo, 1984). The values of n and D_c^k are endogenously determined. To return to their starting point the buses retrace an average of $(L^k+W^k)/2$ miles at zV_f miles per hour and J^k miles at zV_f miles per hour.

d) Buses operate on preset schedules with flexible routing designed to minimize each tour distance D_c^k .

e) Tour departure headways are equal for all zones in the region and uniform within each period.

BUS OPERATIONS WITHOUT TIMED TRANSFERS

This section explores uncoordinated bus operations. Conventional and flexible bus cost functions are formulated while considering transfer cost functions. For both conventional and flexible bus services, total cost functions consist of bus operating cost, user in-vehicle cost, user waiting cost, user transfer cost. For conventional bus services, an access cost term is also included.

In the following sub sections, cost functions for conventional and flexible bus services are formulated, and values of decision variables including vehicle sizes, the number of zones in each region, headways, and fleet sizes are optimized. For uncoordinated bus operations, the conventional service formulation is solved analytically, while the flexible service formulation is solved with a metaheuristic method, namely a genetic algorithm..

Conventional Bus Formulations and Analytic Optimization

The conventional bus cost formulation includes the bus operating cost, user in-vehicle cost, user waiting cost, user access cost, and user transfer cost.

$$C_{tc,c}^{u} = \sum_{k=1}^{K}\{C_{oc,c}^{u,k} + C_{ic,c}^{u,k} + C_{wc,c}^{u,k} + C_{xc}^{u,k} + C_{fc,c}^{u,k}\} \tag{1}$$

Conventional bus operating cost, $C_{oc,c}^{u,k}$, can be formulated by multiplying unit bus operating cost, B, and the number of zones in region k, $N_c^{u,k}$, and fleet size, $F_c^{u,k}$.

$$C_{oc,c}^{u,k} = B^k \cdot N_c^{u,k} \cdot F_c^{u,k} \tag{2}$$

Fleet size, $F_c^{u,k}$, can be formulated as

$$F_c^{u,k} = \frac{D^k}{V_c \cdot h_c^k} \quad (3)$$

Conventional bus user in-vehicle cost is then formulated as

$$C_{ic,c}^{u,k} = v_v Q^k \frac{M^k}{V_c} \quad (4)$$

Since it is assumed that passengers arrive at the stop randomly and uniformly over time, the waiting time may be estimated as in Welding (1957), Osuna and Newell (1972), and Ting and Schonfeld (2005).

$$w^k = \frac{E(h^k)}{2}\left(1 + \frac{(\sigma^k)^2}{[E(h^k)]^2}\right) \quad (5)$$

Thus, the waiting cost of uncoordinated conventional bus service is

$$C_{wc,c}^{u,k} = v_w Q^k w^k = v_w Q^k \frac{E(h_c^k)}{2}\left(1 + \frac{(\sigma^k)^2}{[E(h_c^k)]^2}\right) \quad (6)$$

Since the spacing between adjacent branches of local bus service is r^k, and because service trip origins (or destinations) are uniformly distributed over the area, the average access distance to the nearest route is one-fourth of route spacings, $r^k/4$. Similarly, the access distance alongside the route to the nearest transit stop is one-fourth of the bus stop spacing, d/4. Therefore, the access cost for the conventional bus system, $C_{xc}^{u,k}$, is

$$C_{xc}^{u,k} = \frac{v_x \cdot Q^k (r^k + d)}{4V_x} = \frac{v_x \cdot Q^k (\frac{W^k}{N_c^{u,k}} + d)}{4V_x} \quad (7)$$

Transfer cost, $C_{fc,c}^{u,k}$, can be formulated similarly to the waiting cost function, the only difference being that the transfer cost only considers the transferring passengers:

$$C_{fc,c}^{u,k} = v_f Q_t^k w^k = v_f Q_t^k \left(\frac{E(h_c^k)}{2} + \frac{(\sigma^k)^2}{2E(h_c^k)}\right) \quad (8)$$

For simplicity, the expected value of headway, $E(h^k)$, and the headway, h^k, are interchangeable in this paper. The total cost function of conventional bus service is then

$$C_{tc,c}^{u,k} = B^k \cdot N_c^{u,k} \cdot F_c^{u,k} + v_v Q^k \frac{M^k}{V_c} + v_w Q^k \frac{E(h^k)}{2}\left(1 + \frac{(\sigma^k)^2}{[E(h^k)]^2}\right) + \frac{v_x \cdot Q^k\left(\frac{W^k}{N_c^{u,k}} + d\right)}{4V_x} + v_f Q_t^k \left(\frac{E(h^k)}{2} + \frac{(\sigma^k)^2}{2E(h^k)}\right) \quad (9)$$

By rearranging equation (9) we obtain

$$C_{tc,c}^{u,k} = B^k \cdot N_c^{u,k} \cdot F_c^{u,k} + v_v Q^k \frac{M^k}{V_c} + \frac{h_c^k}{2}\left(v_w Q^k + v_f Q_t^k\right) + \frac{\sigma^2}{2h_c^k}\left(v_w Q^k + v_f Q_t^k\right) + \frac{v_x \cdot Q^k\left(\frac{W^k}{N_c^{u,k}} + d\right)}{4V_x} \quad (10)$$

Equation (3), which is $h_c^k = \frac{D^k}{V_c \cdot F_c^{u,k}}$, is then used to substitute the headway into the fleet size. Equation (11) is now used for optimizing decision variables, namely the number of zones, $N_c^{u,k}$, and fleet sizes, $F_c^{u,k}$.

$$C_{tc,c}^{u,k} = B^k \cdot N_c^{u,k} \cdot F_c^{u,k} + \frac{v_v Q^k M^k}{V_c} + \frac{D^k\left(v_w Q^k + v_f Q_t^k\right)}{2V_c \cdot F_c^{u,k}} + \frac{\sigma^2 V_c \cdot F_c^{u,k}\left(v_w Q^k + v_f Q_t^k\right)}{2D^k} + \frac{v_x \cdot Q^k}{4V_x}\left(\frac{W^k}{N_c^{u,k}} + d\right) \quad (11)$$

By taking the partial derivative of $TC_c^{u,k}$ with respect to the number of zones, $N_c^{u,k}$ we find

$$\frac{\partial C_{tc,c}^{u,k}}{\partial N_c^{u,k}} = B^k F_c^{u,k} - \frac{v_x \cdot Q^k W^k}{4V_x\left(N_c^{u,k}\right)^2} = 0 \quad (12)$$

To guarantee the global minimum solution, we must check the second order derivative of $C_{tc,c}^{u,k}$ with respect to the number of zones, $N_c^{u,k}$. As shown in equation (13), the value of second order derivative is always positive so that the optimal value of the number of zones results in the global minimum.

$$\frac{\partial^2 C_{tc,c}^{u,k}}{\partial (N_c^{u,k})^2} = \frac{v_x \cdot Q^k W^k}{2V_x\left(N_c^{u,k}\right)^3} > 0 \quad (13)$$

After rearranging equation (12), we find

$$(N_c^{u,k})^2 = -\frac{v_x \cdot Q^k W^k}{4B^k V_x F_c^{u,k}} \quad (14)$$

The optimal fleet size is found through analytic optimization. Similarly to the number of zones, the partial derivative of equation (11) with respect to the fleet size, $F_c^{u,k}$, is shown in equation (15).

$$\frac{\partial C_{tc,c}^{u,k}}{\partial F_c^{u,k}} = B^k \cdot N_c^{u,k} - \frac{D^k\left(v_w Q^k + v_f Q_t^k\right)}{2V_c \cdot \left(F_c^{u,k}\right)^2} + \frac{\sigma^2 V_c \cdot \left(v_w Q^k + v_f Q_t^k\right)}{2D^k} = 0 \quad (15)$$

The second order derivative of equation (6.11) with respect to the fleet size, $F_c^{u,k}$, is then

$$\frac{\partial^2 C_{tc,c}^{u,k}}{\partial (F_c^{u,k})^2} = \frac{D^k\left(v_w Q^k + v_f Q_t^k\right)}{V_c \cdot \left(F_c^{u,k}\right)^3} > 0 \quad (16)$$

As shown in equations (14) and (16), the values of second order derivatives are always positive. Since our objective is to minimize the system cost (i.e., equation (11)), optimized values of the number of zones, $N_c^{u,k}$ and fleet size, $F_c^{u,k}$ are guaranteed to yield the global optimum.

By rearranging equation (15), we obtain

$$\left(F_c^{u,k}\right)^2 = \frac{(D^k)^2\left(v_w Q^k + v_f Q_t^k\right)}{V_c\left\{2D^k N_c^{u,k} + \sigma^2 V_c \cdot \left(v_w Q^k + v_f Q_t^k\right)\right\}} \quad (17)$$

the square of equation (14) is expressed in equation (18) as follows:

$$\left(F_c^{u,k}\right)^2 = \frac{(v_x)^2 \cdot (Q^k)^2 (W^k)^2}{16(B^k)^2 (V_x)^2 (N_c^{u,k})^4} \tag{18}$$

Now, equations (17) and (18) are set to be equal, and arranged as a function of the number of zones, $N_c^{u,k}$.

$$16(B^k)^2(V_x)^2(N_c^{u,k})^4 - 2(v_x)^2 D^k B^k (Q^k)^2 (W^k)^2 V_c N_c^{u,k} - \sigma^2 (v_x)^2 (Q^k)^2 (W^k)^2 (V_c)^2 \left(v_w Q^k + v_f Q_t^k\right) = 0 \tag{19}$$

Equation (19) is a fourth order polynomial equation. The values of the number of zones, $N_c^{u,k}$can be found with a MATLAB function called "roots". This "roots" function solves polynomial equations with eigenvalues of the companion matrix. Details are provided in Mathworks (2013). The number of zones, which is obtained from the "roots" function, may include negative or imaginary values. However, only positive real numbers are chosen and rounded to the nearest integer values. Then, the value of the number of zones, $N_c^{u,k}$ is substituted into equation (14) to find the optimized fleet size, $F_c^{u,k}$. Since both of these variables have to be integer, the nearest integer from the optimized value is chosen for both the fleet size and the number of zones.

The minimum cost headway can be obtained from the optimized fleet size, $F_c^{u,k}$, with equation (3), which can be rewritten as follows:

$$h_c^{k,min} = \frac{D^k}{V_c \cdot F_c^{u,k}} \tag{20}$$

The resulting headway should not exceed the maximum allowable headway, which is expressed in equation (21):

$$h_c^{k,max} = \frac{S^k l^k N_c^{u,k}}{f^k \cdot Q^k}$$

(21)

In the conventional bus service formulations above, i.e., equations (1) ~ (21), the vehicle size affects the system performance characteristics such as bus operating cost and maximum allowable headways. However, the vehicle size is treated as an input value, rather than optimized. In this paper, the following list of available vehicle sizes is considered: {7, 10, 16, 25, 35, 45 seats}. By simply analyzing equations repeatedly for each available vehicle size, the minimum cost vehicle size is found for each region.

Flexible Bus Formulations and Numerical Optimization

The flexible bus cost formulation considers bus operating cost, user in-vehicle cost, user waiting cost, and user transfer cost. Unlike in the conventional bus formulation, the user access cost is not included because flexible bus services are assumed here to pick up passengers and drop them at their doorsteps.

$$C_{tc,f}^{u} = \sum_{k=1}^{K}\{C_{oc,f}^{u,k} + C_{ic,f}^{u,k} + C_{wc,f}^{u,k} + C_{fc,f}^{u,k}\} \quad (22)$$

The flexible bus operating cost in region k, $C_{oc,f}^{u,k}$, is formulated by multiplying bus operating cost, B^{k}, by the number of flexible service zones , $N_f^{u,k}$, and the fleet size, $F_f^{u,k}$.

$$C_{oc,f}^{u,k} = B^{k} \cdot N_f^{u,k} \cdot F_f^{u,k} \quad (23)$$

The fleet size $F_f^{u,k}$ is the total flexible bus travel time divided by the headway. The total flexible bus travel distance is the sum of the average travel distance between the terminal and the local region, D_f^{k}, and the flexible bus tour distance in the region, D_c^{k}.

$$F_f^{u,k} = \frac{(D_f^k + D_c^k)}{V_f h_f^k} \quad (24)$$

Following Stein (1978), the flexible bus tour distance, D_c^{k}, is then formulated as:

$$D_c^k = \emptyset\sqrt{nA^k} \quad (25)$$

The number of passengers in one tour, n, is shown in equation (26) below:

$$n = \frac{A^k Q^k h_f^k}{u L^k W^k} \quad (26)$$

By substituting equation (26) and $A^k = L^k W^k / N_f^{u,k}$ into equation (25), the approximated flexible bus tour distance in region k is:

$$D_c^k = \frac{\emptyset}{N_f^{u,k}}\sqrt{\frac{L^k W^k Q^k h_f^k}{u}} \quad (27)$$

By substituting equations (24) and (27) into equation (23), the flexible bus operating cost is formulated as follows:

$$C_{oc,f}^{u,k} = \frac{D_f^k B^k N_f^{u,k}}{V_f h_f^k} + \frac{\emptyset B^k}{V_f}\sqrt{\frac{L^k W^k Q^k}{u h_f^k}} \quad (28)$$

The flexible bus in-vehicle cost is then formulated as follows:

$$C_{ic,f}^{u,k} = \frac{v_v Q^k (D_f^k + D_c^k)}{2V_f} = \frac{v_v Q^k D_f^k}{2V_f} + \frac{\emptyset v_v Q^k}{2V_f N_f^{u,k}}\sqrt{\frac{L^k W^k Q^k h_f^k}{u}} \quad (29)$$

As mentioned earlier regarding equation (5), it is assumed that passengers arrive at the terminal randomly and uniformly over time. Therefore, the waiting time in equation (5) is still applicable to flexible bus services. Thus the waiting cost of flexible bus service is

$$C_{wc,f}^{u,k} = v_w Q^k w^k = v_w Q^k \frac{E(h_f^k)}{2}\left(1 + \frac{(\sigma^k)^2}{\left[E(h_f^k)\right]^2}\right) \tag{30}$$

The flexible bus transfer cost function is also formulated similarly:

$$C_{fc,f}^{u,k} = v_f Q_t^k w^k = v_w Q_t^k \frac{E(h_f^k)}{2}\left(1 + \frac{(\sigma^k)^2}{\left[E(h_f^k)\right]^2}\right) \tag{31}$$

Similarly to conventional bus formulations, the expected value of headway, $E(h_f^k)$, and the headway, h_f^k, are assumed here to be interchangeable.

By substituting equations (28) ~ (31), into equation (22), the total flexible bus cost in region k becomes

$$C_{tc,f}^{u,k} = \frac{D_f^k B^k N_f^{u,k}}{V_f h_f^k} + \frac{\emptyset B^k}{V_f}\sqrt{\frac{L^k W^k Q^k}{u h_f^k}} + \frac{v_v Q^k D_f^k}{2V_f} + \frac{\emptyset v_v Q^k}{2V_f N_f^{u,k}}\sqrt{\frac{L^k W^k Q^k h_f^k}{u}} + \frac{v_w Q^k h_f^k}{2} + \frac{v_w Q^k (\sigma^k)^2}{2h_f^k} + \frac{v_f Q_t^k h_f^k}{2} + \frac{v_f Q_t^k (\sigma^k)^2}{2h_f^k} \tag{32}$$

Equation (32) is a function of three decision variables, namely the vehicle size, the number of zones, $N_f^{u,k}$, and the headway, h_f^k. The fleet size can be found from the headway. Since analytic optimization of equation (32) is difficult, a numerical method (Real Coded Genetic Algorithm (RCGA)) is chosen to optimize decision variables, namely the vehicle size, the number of zones, and fleet sizes for each region. The resulting minimum cost headway can be found by substituting equation (27) into equation (24):

$$F_f^{u,k} = \frac{D_f^k}{V_f h_f^k} + \frac{\emptyset}{V_f N_f^{u,k}}\sqrt{\frac{L^k W^k Q^k}{u h_f^k}} \tag{33}$$

After rearranging equation (33) as a function of the headway by setting t equal to $\sqrt{\frac{1}{h_f^k}}$, we obtain:

$$\frac{D_f^k}{V_f} t^2 + \frac{\emptyset}{V_f N_f^{u,k}}\sqrt{\frac{L^k W^k Q^k}{u}}\, t - F_f^{u,k} = 0 \tag{34}$$

Thus, t has two values (one positive and one negative) that satisfy equation (34). The higher (positive) value of t is chosen.

$$t = \left\{ -\frac{\emptyset}{V_f N_f^{u,k}}\sqrt{\frac{L^k W^k Q^k}{u}} + \sqrt{\left(\frac{\emptyset}{V_f N_f^{u,k}}\right)^2 \sqrt{\frac{L^k W^k Q^k}{u}} + 4F_f^{u,k}\frac{D_f^k}{V_f}} \right\} \Bigg/ \frac{2D_f^k}{V_f} \tag{35}$$

The optimized headway that ensures integer number of zones, $N_f^{u,k}$, and the fleet size, $F_f^{u,k}$, is now given by equation (36). The optimized minimum cost headway should be less than or equal to the maximum allowable headway, $h_f^{k,max} = \frac{S^k l^k N_f^{u,k}}{\cdot Q^k}$.

$$h_f^{k,min} = \frac{4\left(D_f^k\right)^2}{(v_f)^2\left\{-\frac{\varpi}{v_f N_f^{u,k}}\sqrt{\frac{L^k W^k Q^k}{u}}+\sqrt{\left(\frac{\varpi}{v_f N_f^{u,k}}\right)^2\sqrt{\frac{L^k W^k Q^k}{u}}+4F_f^{u,k}\frac{D_f^k}{v_f}}\right\}^2} \tag{36}$$

The total cost is sum of the costs for individual regions. Thus, the RCGA independently optimizes the vehicle size, the number of zones, and fleet sizes for each region. All decision variables are integer. For the vehicle size, a list of vehicle sizes is considered (e.g., 7, 10, 16, 25, 35, 45 seats). For instance, x is the indicator of the vehicle size variable. If x = 1, then the selected vehicle size is 7 seats. If x = 2, then the vehicle size is 10. Therefore, x is integer and it ranges from one to six. A graphical representation of the solution method for the uncoordinated flexible service is shown in Figure 3. For uncoordinated flexible bus operations, the system-wide cost is the sum of the regional costs, which may be independently optimized.

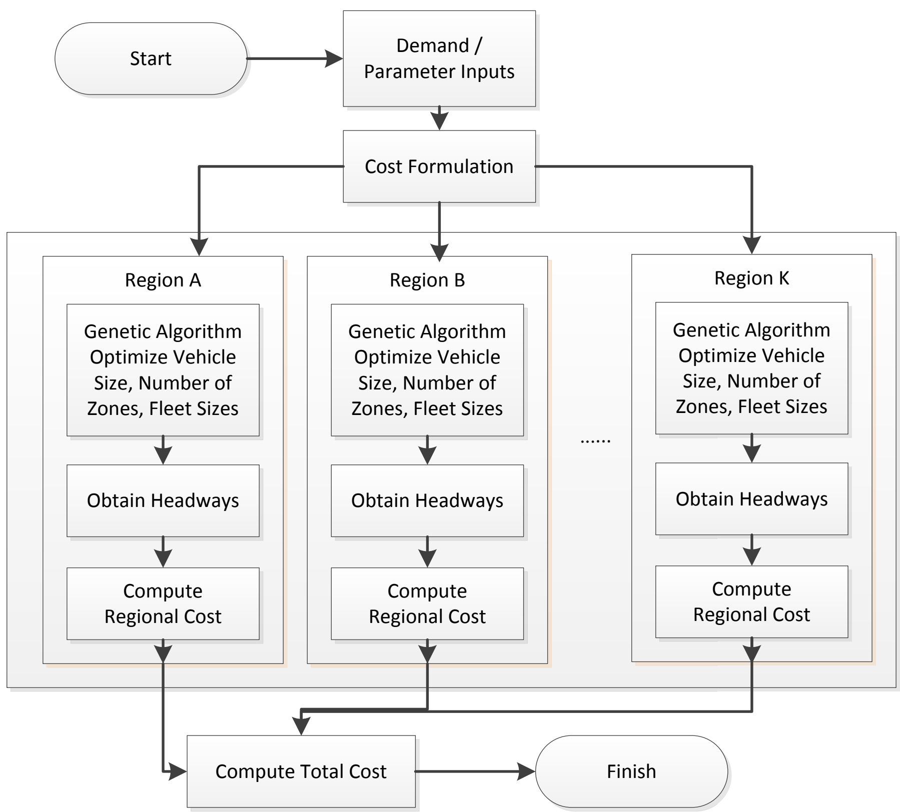


Figure 3 Solution Method for Uncoordinated Flexible Services

Numerical Examples for Uncoordinated Bus Operations

In this section we provide numerical examples to explore the threshold between conventional and flexible services, for uncoordinated bus operations. The developed models for both conventional and flexible services provide solutions for one time period. Therefore, the total cost for the system is simply the sum of total cost for each region. The solutions for conventional services are analytically optimized while the solutions for flexible services are found with the RCGA metaheuristic. Since RCGA is an iterative search technique and does not guarantee the global optimality of solutions, we consider a system with four local regions for our numerical examples. We find that solutions obtained with RCGA are identical among regions. This confirms that RCGA provides consistent solutions. For sensitivity analyses, we explore demand, line haul distance, length of region, width of region, and operating cost parameters. Table 2 shows the demand inputs. The unit of demand input is trips per minute. These numbers of trips are assumed to be uniformly distributed in the target region. The other baseline values of parameters are provided in Table 1. We also note that all standard deviations of bus arrivals are assumed to be 0.5 minutes. Table 3 provides the baseline inputs for line haul distances, lengths and widths of regions.

Table 2 Demand Inputs

	A	B	C	D	Terminal
A	0.0	0.2	0.2	0.2	0.2
B	0.2	0.0	0.2	0.2	0.2
C	0.2	0.2	0.0	0.2	0.2
D	0.2	0.2	0.2	0.0	0.2
Terminal	0.2	0.2	0.2	0.2	0.0

Table 3 Regional Inputs

	A	B	C	D
Line-haul Distance	1	1	1	1
Length of Region	4	4	4	4
Width of Region	3	3	3	3

Results for the sensitivity of the total cost of the system with respect to line-haul distance, length of regions are provided in Table 4. Since we consider four identical local regions, the total cost of each region is the total system cost divided by the number of regions (i.e., four). To shorten this paper we combine two sensitivity analyses results into Table 4. We find that flexible services are preferable to conventional services for line-haul distances up to four miles. The thresholds between conventional and flexible services are boldly marked and underlined in Table 4. It should be noted that threshold analysis between conventional and flexible services which includes the transfer cost term has not been found in any previous studies. We then analyze the effect of region length on total system cost. Results in Table 4 show that the threshold for the length of the region between conventional and flexible services is between five and six miles. Shorter regions favor flexible services. As expected, sensitivity analyses of line-haul distance and length of region confirm that when the round trip distance (or time) increases, conventional services are preferred because they can transport more passengers per round trip than flexible services.

Table 4 Sensitivity Analysis Results for Line-haul and Length of Regions

		Conventional Service Regional Cost	Conventional Service Total Cost	Flexible Service Regional Cost	Flexible Service Total Cost	Service Type
J (miles)	1	12.31	49.24	11.95	47.82	Flex
	2	12.91	51.65	12.74	50.96	Flex
	3	**13.52**	**54.07**	**13.51**	**54.03**	**Flex**
	4	**14.12**	**56.49**	**14.28**	**57.10**	**Conv**
	5	14.73	58.90	14.92	59.68	Conv
L (miles)	1	9.11	36.46	7.70	30.78	Flex
	2	10.49	41.95	9.49	37.96	Flex
	3	11.49	45.96	10.70	42.80	Flex
	4	12.31	49.24	11.95	47.82	Flex
	5	**13.13**	**52.52**	**13.09**	**52.37**	**Flex**
	6	**13.95**	**55.80**	**14.07**	**56.26**	**Conv**
	7	14.77	59.09	14.88	59.54	Conv
	8	15.57	62.29	15.64	62.56	Conv
	9	16.21	64.83	16.40	65.58	Conv
	10	16.84	67.37	17.24	68.96	Conv

We also analyze the sensitivity to demand, which is shown in Table 5. Conventional services are preferable to flexible services at higher demand, in terms of the total cost. The demand threshold between conventional and flexible services is about 0.4 ~ 0.5 trips per minute, given our other inputs.

Table 5 Sensitivity Analysis Results for Demand Inputs

	Q	Conventional Service Regional Cost	Conventional Service Total Cost	Flexible Service Regional Cost	Flexible Service Total Cost	Service Type
Q (trips/min)	0.1	9.04	36.17	8.49	33.95	Flex
	0.2	14.66	58.64	14.39	57.56	Flex
	0.3	19.74	78.96	19.63	78.52	Flex
	0.4	**24.61**	**98.43**	**24.42**	**97.66**	**Flex**
	0.5	**28.83**	**115.33**	**29.14**	**116.56**	**Conv**
	0.6	33.15	132.59	33.61	134.43	Conv
	0.7	37.23	148.90	37.94	151.76	Conv
	0.8	41.20	164.78	42.07	168.28	Conv
	0.9	45.04	180.18	46.27	185.07	Conv
	1	48.89	195.58	50.15	200.61	Conv

Table 6 combines two analyses for time related factors. We explore the effect of in-vehicle and access time on the system. The values of in-vehicle and access times in Table 6 are shown in $/hour, but they are applied in $/minute for analyses. When the value of the in-vehicle time is high, conventional services are chosen over flexible services. Table 4 provides results for line-haul distance and length of region. We find that conventional services are the selected service type over flexible services for the longer round travel times. Table 6 also shows the effects of the access time factor. Since flexible services are assumed to provide door-to-door services, we analyze conventional services and find what value of access time for conventional services becomes the threshold between conventional and flexible services. This threshold access time value is around 12.5 ~ 15.0 $/ hour. It is shown in Figure 4 (f), as approximately 13.5 $/ hour.

Table 6 Sensitivity Analysis Results for Value of Time Inputs

		Conventional Service Regional Cost	Conventional Service Total Cost	Flexible Service Regional Cost	Flexible Service Total Cost	Service Type
v_v ($/hour)	5	11.41	45.62	9.96	39.85	Flex
	7.5	11.86	47.43	10.96	43.83	Flex
	10	**12.31**	**49.24**	**11.95**	**47.82**	**Flex**
	12.5	**12.76**	**51.05**	**12.80**	**51.21**	**Conv**
	15	13.21	52.85	13.64	54.55	Conv
	17.5	13.67	54.66	14.47	57.89	Conv
	20	14.12	56.47	15.16	60.63	Conv
v_x ($/hour)	5	8.94	35.76	11.95	47.82	Conv
	7.5	10.22	40.88	11.95	47.82	Conv
	10	10.95	43.80	11.95	47.82	Conv
	12.5	**11.63**	**46.52**	**11.95**	**47.82**	**Conv**
	15	**12.31**	**49.24**	**11.95**	**47.82**	**Flex**
	17.5	12.99	51.96	11.95	47.82	Flex
	20	13.67	54.68	11.95	47.82	Flex

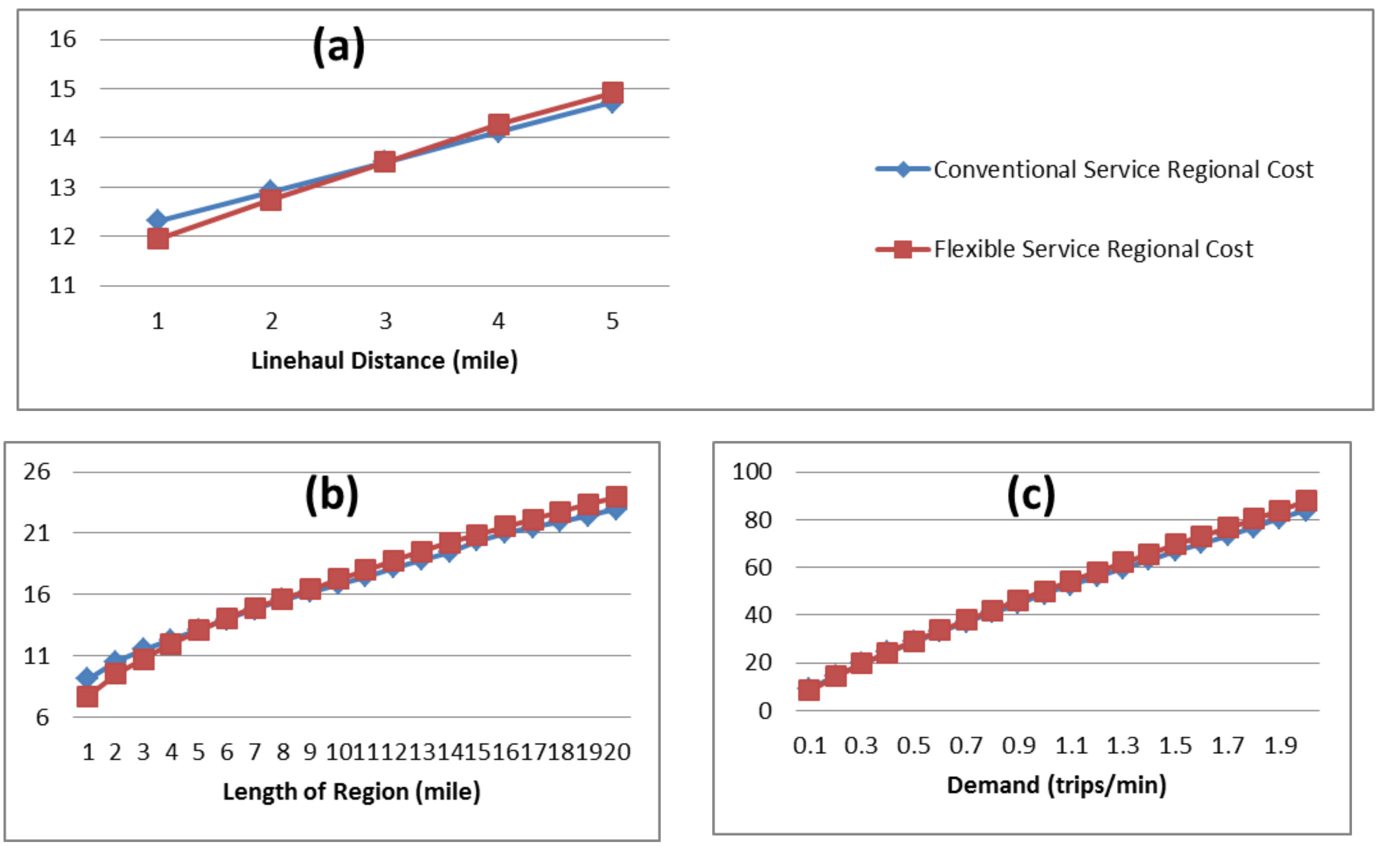
The operating cost is also a significant factor in determining choice between conventional and flexible services. For the operating cost function, we consider two parameters (a and b) in the operating cost function, which is B = a + bS. Table 7 shows the threshold between conventional and flexible services. As expected, a high unit operating cost favors conventional services over flexible services.

Table 7 Sensitivity Analysis Results for Operating Cost Parameters

		Conventional Service Regional Cost	Conventional Service Total Cost	Flexible Service Regional Cost	Flexible Service Total Cost	Service Type
a ($/hour)	20	9.82	39.29	8.10	32.41	Flex
	25	10.32	41.29	8.85	35.41	Flex
	30	10.82	43.29	9.53	38.12	Flex
	35	11.31	45.24	10.20	40.78	Flex
	40	11.64	46.57	10.86	43.45	Flex
	45	11.98	47.90	11.45	45.82	Flex
	50	12.31	49.24	11.95	47.82	Flex

	55	12.64	50.57	12.45	49.82	Flex
	60	**12.98**	**51.90**	**12.95**	**51.82**	**Flex**
	65	**13.31**	**53.24**	**13.45**	**53.82**	**Conv**
	70	13.64	54.57	13.96	55.83	Conv
	0.2	12.31	49.24	11.95	47.82	Flex
	0.4	12.40	49.61	12.11	48.42	Flex
	0.6	12.50	49.98	12.25	48.98	Flex
	0.8	12.59	50.36	12.39	49.54	Flex
b ($/seat.hr)	1	12.68	50.73	12.53	50.10	Flex
	1.2	12.78	51.10	12.67	50.66	Flex
	1.4	12.87	51.48	12.81	51.22	Flex
	1.6	**12.96**	**51.85**	**12.95**	**51.78**	**Flex**
	1.8	**13.06**	**52.22**	**13.09**	**52.34**	**Conv**
	2	13.15	52.60	13.23	52.90	Conv

Figure 4 provides graphical representations of thresholds. To save space we combine eight graphs in Figure 4. The units for vertical axis ($/minute) are identical for all graphs. Tables 4~7 provide both the total system cost and the regional cost. We use the regional cost to compare costs graphically in Figure 4.



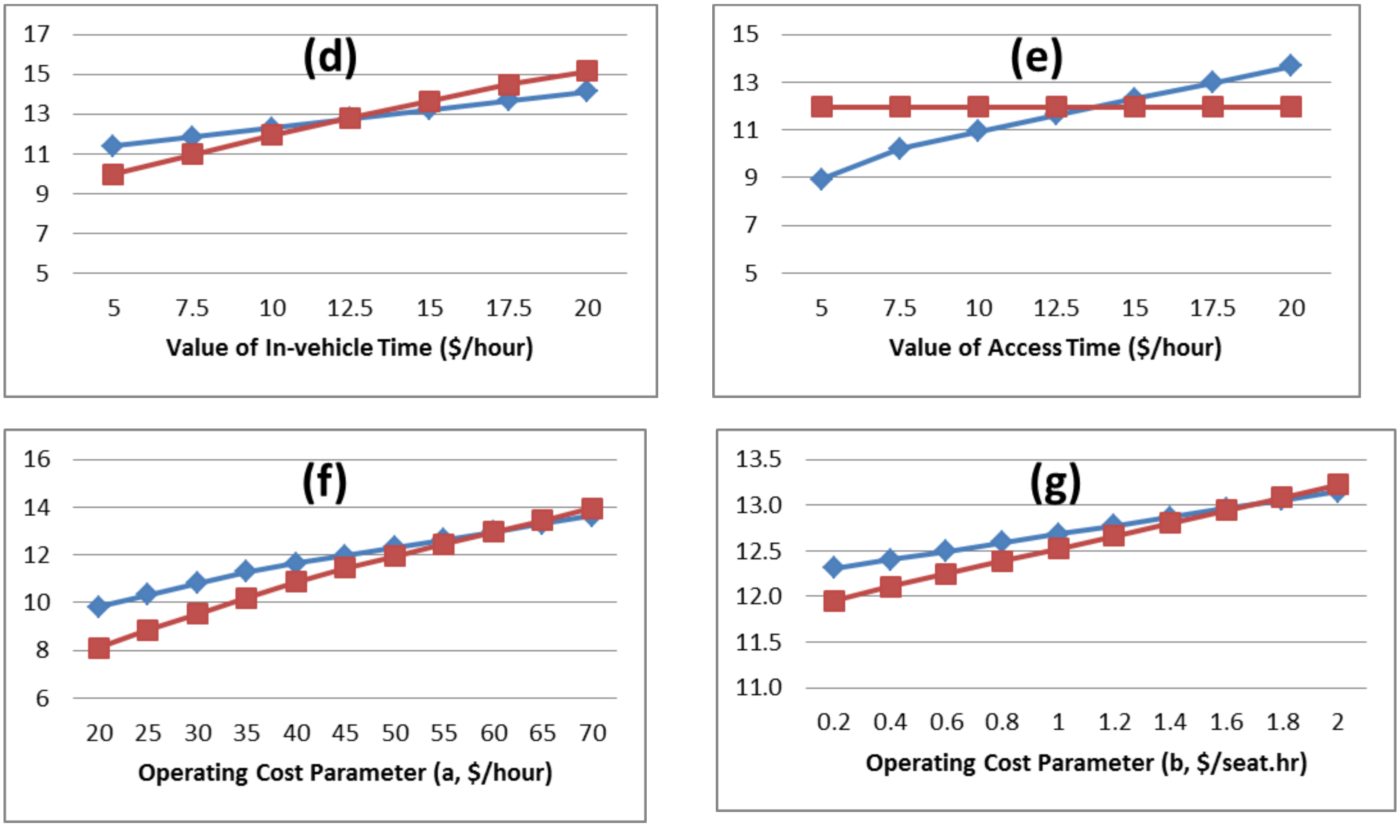


Figure 4 Sensitivity Analyses for Uncoordinated Operations, (a) ~ (g)

With sensitivity analyses of important parameters, we find the thresholds between conventional and flexible services. The results up to this point are analyzed without timed transfer coordination. The next section provides the formulation of the cost for timed transfers.

FORMULATION FOR TIMED TRANSFERS COORDINATION

For coordinated passenger transfers, the provision of a buffer (slack) time may be desirable for increasing the probability of successful vehicle connections at the transfer terminal. By providing slack times, the probabilities that passengers miss their connecting vehicles at the terminal are decreased. However, an additional cost is incurred for vehicle operations and travel times for passengers already on board when slack times are provided. For timed transfers, we optimize headways and resulting fleet as integer values. For finding integer values for headways (either common headway or integer multiple headway) and resulting fleet with a given round travel time, the slack time becomes a dependent variable. Slack times also help achieve headways that are multiples of a base cycle, thereby helping synchronize vehicle arrivals at the main transfer terminal. For timed passenger transfers, the transfer cost consists of the induced slack cost, inter-cycle waiting cost, missed connection cost and delayed connection cost. Thus, the transfer cost for timed transfer operations is the sum of equations (37) and (39) ~ (41). For other costs, namely the operating, in-vehicle, waiting, and access cost, are identical to formulations for uncoordinated operations, but those costs are modified with integer multiple headways and resulting fleet sizes.

Induced Slack Cost

The slack time cost FC_s is an additional cost that induced by a slack time. The slack time cost is formulated as:

$$C_s \;= \sum_k (v_f Q_{nt}^k + \frac{B_k N_k}{h_k})\, s_k \tag{37}$$

where Q_{nt}^k is the sum of non-transfer demand (= terminal to region k + region k to terminal) in region k; s_k is the slack time to region k at the terminal in minute. The first term is the waiting cost of non-transferring passengers to and from region k, and the second term is the increased operating cost due to the slack time.

Inter-cycle Waiting Cost

The relation for pairs of routes with headways y and 2y is shown in Figure 5. The inter-cycle waiting times between connecting regions j and k are expressed in equation (38). It should be noted that if the common headway is chosen as the solution, the first term is canceled out and the waiting time is equal to the slack time of that region. The inter-cycle waiting cost C_i is a cost summation for all regions that are connected at the terminal, as expressed in equation (39):

$$z_{jk} \;= g_{jk}\, y \left(\frac{h_k}{2 g_{jk} y} - \frac{1}{2}\right) + s_k \tag{38}$$

where z_{jk} are the transfer waiting times from region j to region k that are caused by the inter-cycle headways; y is the base cycle; and g_{jk} is the greatest common divisor of β_j and β_k.

$$C_i \;= \sum_j \sum_{\substack{k \\ j \neq k}} v_f Q_{jk}\, s_k \tag{39}$$

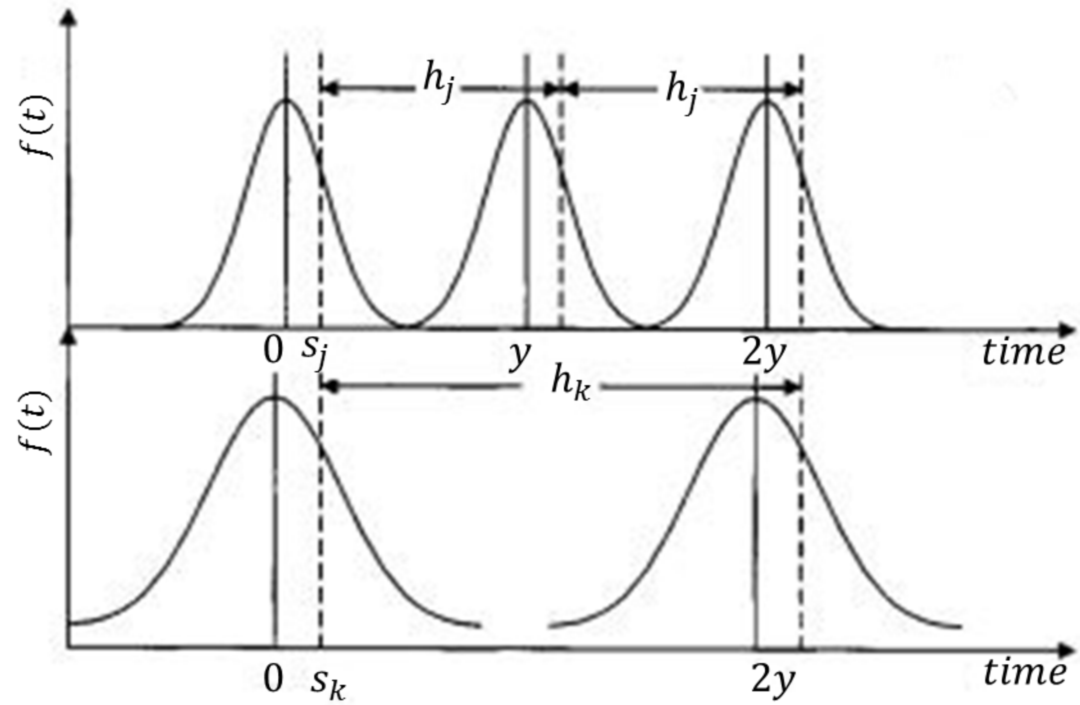


Figure 5 Inter-cycle Waiting Time

Missed Connection Cost

The probability of missed connection cost C_m includes two possible cases: 1) the feeder vehicle arrives late while the receiving vehicle is not late; and 2) both vehicles are late, but feeder vehicle arrives after the receiving vehicle leaves. Therefore, the missed connection delay is the cost of waiting during one additional headway. The graphical representation of waiting time due to the missed connection and the delayed connection can be found in Ting and Schonfeld (2005).

The missed connection cost is formulated as:

$$C_m = \sum_j \sum_{\substack{k \\ j\neq k}} v_f Q_{jk} \left[\int_{s_j}^{h_j} \int_{-h_k}^{s_k} (h_k - s_k - t_j + s_j) f(t_k) dt_k f(t_j) dt_j + \int_{s_j}^{h_j} \int_{s_k}^{t_j - s_j + s_k} (h_k - s_k - t_j + s_j) f(t_k) dt_k f(t_j) dt_j\right] \tag{40}$$

where Q_{jk} is transfer demand from region j to k; s_j is slack time for region j at the terminal; t_j is the distribution of vehicle arrival times in region j; $f(t_j)$ is the probability density function of arrival times in region j.

Delayed Connection Cost

The delayed connection cost considers two possible cases: 1) the feeder vehicle arrives early, but the receiving vehicle is late; 2) both vehicles are late, but the feeder vehicle arrives before the receiving vehicle. The delayed connection cost FC_d is formulated as

$$C_d = \sum_j \sum_{\substack{k \\ j \neq k}} v_f Q_{jk} \left[\int_{-h_j}^{s_j} \int_{s_k}^{h_k} (h_k - s_k) f(t_k) d t_k f(t_j) d t_j + \int_{s_j}^{h_j} \int_{t_j - s_j + s_k}^{h_k} (t_k - s_k - t_j + s_j) f(t_k) d t_k f(t_j) d t_j \right] \quad (41)$$

Solution Method

For timed transfers operation, we re-optimize headways and resulting fleet size. Other decision variables, which are the service type selections, optimized vehicle size(s) and the number of zone(s) for both conventional and flexible services, are adopted from the optimization results of uncoordinated operations. As explained earlier, slack times are desirable for increasing the probability of vehicle connections at the terminal and for ensuring an integer bus fleet.

We assume that the headways to be optimized are integer multiple(s) of the base cycle. For instance, the headway of region k is expressed as $h_k = \beta_k y$ where y is the base cycle and β_k is an integer number. β_k is a decision variable for the headway of the region k and y is a decision variable applicable for all regions. If we need to optimize headways for n local regions, we have n+1 decision variables. For realism, we assume that the value of the base cycle, y, is chosen among {2, 3, 4, 5, 6, 7.5, 10, 12, 15, 20 and 30 minutes}. If β_k has the value of one for all regions, this means that all regions are synchronized with the common headway, which is y. For example, a possible case of optimized integer multiple headways is that the base cycle, y, of three minutes and β_k with 2, 3, 3 and 3 cycles for regions A, B, C, and D. In this case, the resulting headways are 6, 9, 9, and 9 minutes for regions A, B, C, and D. To find headways and resulting fleet size, we use RCGA. Details on RCGA can be found in Kim and Schonfeld (2013), Deb (2000), and Deep et al. (2009). In the RCGA solution search procedure we also ensure the optimized solution h_k does not exceed the maximum allowable headway for region k.

NUMERICAL ANALYSIS

In this numerical analysis section, we explore the applicability of the timed transfer formulation. From the numerical analyses of uncoordinated operations, we find that the integration of conventional and flexible services reduces the total cost. By providing timed at the terminal, it may be possible to further reduce the cost. We compare several cases with the costs of uncoordinated operations and timed transfers operations. As mentioned earlier, we first optimize uncoordinated operation solutions, namely the service type, vehicle size, headway, fleet, number of zones in region. For the solution of timed transfers, we re-optimize headways and resulting fleet size, but solutions for other decision variables can be adopted from the solution for uncoordinated operations.

Case I: Integer Multiple Headway Solutions

For the first case, we consider four regions with different regional characteristics. Each region has different line haul distance, thus the route travel times vary by regions. We also consider different lengths and widths of regions. Table 8 provides input values for demand, line-haul distance, length of region, width of region, and standard deviation of vehicle arrivals. It should also be noted that vehicle arrival variances are assumed to differ among regions, as shown in Table 8.

Table 8 Demand and Other Inputs

Demand (trips/min)					
Region / Period	A	B	C	D	T
A	0.00	0.11	0.14	0.02	0.01
B	0.22	0.00	1.33	1.33	0.44
C	0.69	1.39	0.00	0.69	0.56
D	0.02	0.56	0.11	0.00	0.01
T	0.03	0.11	0.17	0.02	0.00
Region	A	B	C	D	
Line-haul Distance (miles)	3.5	8.5	6	5	
Length of Region (miles)	2	6	4	3	
Width of Region (miles)	4	4	4	6	
Standard Deviation of Vehicle Arrivals (min)	0.5	1	0.3	0.5	

Uncoordinated Operation Results in Case 1

Here, we first discuss the results of uncoordinated operations, with the inputs given in Table 8. Other baseline values, which are needed but not shown in Table 8, are as shown in Table 1. Regions A and D are chosen to provide flexible services while regions B and C are served by conventional services. The line haul distance J is one of factors that affect the service type selection. We consider in-vehicle cost in our formulations, both conventional and flexible services. J is a major determinant of the vehicle round trip time, and hence round trip cost, between the terminal and local region, so that it can significantly affect to the service type selection. However, it should be noted that the service type decision is determined along with other factors, such as demand density and cost factors. In the optimization process vehicle sizes are selected at either 10 seats or 16 seats per bus; here 10 seat buses are selected for regions A and C while 16 seats buses are selected for regions B and D. Table 9 also provides details on the optimized number of zones for each region, optimized headways, resulting fleet size, and costs. As expected, the total costs of region A and D, in which flexible services are selected for operations, are lower than the total costs of conventional services in region B and C. In Table 8, the line-haul distances for regions B and C are longer than those for regions A and D. Longer line-haul distances imply longer round travel times. Thus, the solution we find also confirms that conventional services are preferable to flexible services when round travel times are long while other factors are fixed. The optimized headways are about eight to twelve minutes. Required fleet size varies from four to five buses per zone. The number of zones is one for region A and three for regions B, C, and D. The total cost of uncoordinated operations is 130.49 $/minute. It should be noted that optimized headways need not be rounded to integer values for uncoordinated operations.

Table 9 Uncoordinated Operation Results in Case 1

	Regions				sum
	A	B	C	D	
Service Type*	2	1	1	2	-
Vehicle Size (seats)	10	16	10	16	-
Number of Zones	1	3	3	3	-
Headway (min)	8.05	11.83	8.44	11.10	-
Fleet Size Per Zone (buses)	5	4	4	4	-
Operating Cost ($/min)	4.33	10.64	10.4	10.64	36.01
In-Vehicle Cost ($/min)	4.16	16.17	10.91	10.21	41.45
Waiting Cost($/min)	0.84	5.45	3.58	2.56	12.43
Access Cost ($/min)	0	12.63	11.68	0	24.31
Transfer Cost ($/min)	1.16	7.06	1.41	3.64	13.27
Total Cost ($/min)	10.49	51.95	40.99	27.06	130.49

* 1: Conventional Services; 2: Flexible Services

Timed Transfers Coordination Results in Case 1

For coordinated transfer costs, equations (37~41) are used. Operating, in-vehicle, waiting, and access costs for conventional and flexible services are the same as for uncoordinated operations. Table 9 confirms that the integration of conventional and flexible services is desirable when demand and other regional characteristics vary over regions. By providing timed transfers for passengers, it may be possible to further reduce the total cost by mainly reducing the cost of passengers' waiting times at transfer stations. We adopt the optimized solutions for the service type, vehicle size, and number of zones. Here we optimize the headway, which can be either a common headway or integer multiple headways for the different regions, and the resulting fleet size. The headways and fleet size are constrained to integer values by adding slack times to the round travel times. The optimized headways are found as multiples of the optimized three minutes base cycle. As shown in Table 10, the headways for regions A, B, C, and D are 6, 12, 9, and 12 minutes, respectively. The resulting optimized headways are not exceeded the maximum (capacity-constrained) allowable headway for each region, as shown in Table 9. The optimized fleets per zone are chosen as 7, 4, 4, and 4 vehicles; thus total fleet is 7, 12, 12, and 12 vehicles for regions A, B, C, and D, respectively. It should be noted that vehicle sizes differ among regions. The resulting fleet size for region A changes from 5 to 7 buses per zone because the optimized headways in region A decrease from 8.06 to 6 minutes. The total cost of timed transfers operations is 126.88 $/minutes, which is reduced by about 3.61 $/minute, or 2.76% = (126.88-130.49)/130.49.

Table 10 Timed Transfers Coordination Results in Case 1

	Regions				sum
	A	B	C	D	
Service Type	2	1	1	2	-
Vehicle Size (seats)	10	16	10	16	-
Number of Zones	1	3	3	3	-
Headway (min)	6	12	9	12	-
Max. Allowable Headway	8.06	14.46	9.01	17.39	-
Slack Time (min)	5.11	0.67	2.22	2.74	-
Fleet Size Per Zone (buses)	7	4	4	4	-
Operating Cost ($/min)	6.07	10.64	10.40	10.64	37.75
In-Vehicle Cost ($/min)	3.81	16.17	10.91	10.41	41.30
Waiting Cost($/min)	0.62	5.53	3.81	2.76	12.73
Access Cost ($/min)	0.00	12.63	11.68	0.00	24.31
Induced Slack Cost ($/min)	0.77	0.21	0.91	0.62	
Inter-cycle Waiting Cost ($/min)					7.38
Missed Connection Cost ($/min)					0.87
Delayed Connection Cost ($/min)					0.03
Transfer Cost ($/min)					10.79
Total Cost ($/min)					126.88

Case 2: Common Headway Solution

For the second case , we increase demand inputs by a factor of five compared to Case 1, as shown in Table 11. For other parameters, baseline values from Table 1 are used.

Input Values for Case 2

Table 11 Demand and Other Inputs

Demand (trips/min)					
Region / Period	A	B	C	D	T
A	0.00	0.55	0.70	0.10	0.05
B	1.10	0.00	6.65	6.65	2.20
C	3.45	6.95	0.00	3.45	2.80
D	0.10	2.80	0.55	0.00	0.05
T	0.15	0.55	0.85	0.10	0.00

Uncoordinated Operation Results in Case 2

By quintupling the demand, the optimized vehicle sizes and the numbers of zones per region are increased, as shown in Table 12. Optimized vehicle sizes are either 16 seats for regions A and D or 25 seats for regions B and C. It is also notable that the conventional service is chosen here for region D while flexible services are selected in Case 1. The numbers of zones per region also increase compared to the results in Table 9. The access cost of region A is zero because flexible services are chosen. Optimized headways vary between 5.63 and 7.44 minutes. With higher demand inputs (compared to Case 1), the optimized headways decrease; thus, the service frequencies increase. The total cost of uncoordinated operations is 469.32 $/minute.

Table 12 Uncoordinated Operation Results in Case 2

	Regions				sum
	A	B	C	D	
Service Type	2	1	1	1	-
Vehicle Size (seats)	16	25	25	16	-
Number of Zones	3	5	5	6	-
Headway (min)	6.38	6.76	5.63	7.44	-
Fleet Size Per Zone (buses)	5	7	6	4	-
Operating Cost ($/min)	13.3	32.08	27.5	21.28	94.16
In-Vehicle Cost ($/min)	16.53	80.83	54.56	27.34	179.26
Waiting Cost($/min)	3.33	15.81	11.95	8.6	39.69
Access Cost ($/min)	0	41.18	38.1	24.84	104.12
Transfer Cost ($/min)	4.63	20.48	14.74	12.25	52.1
Total Cost ($/min)	37.79	190.37	146.85	94.31	469.32

Timed Transfers Coordination Results in Case 2

The optimized headways for timed transfers are found with the common headway solution, which is six minutes. As demand increases, the service frequency also increases, which means smaller optimized headways. If optimized headways are similar to those for other regions, synchronization with the common headway is possible, which is the result of Case 2. The headways for all regions are 6 minutes, which means that vehicles are scheduled to meet every six minutes at the terminal. With the provision of slack times, the operating costs increase while transfer costs decrease, which is the main trade-off for timed transfers at the terminal. The total cost reduction is 19.9 $/minute (4.24 % = (449.42-469.32)/469.32).

Table 13 Timed Transfers Coordination Results in Case 2

	Regions	sum

	A	B	C	D	
Service Type	2	1	1	1	
Vehicle Size (seats)	16	25	25	16	
Number of Zones	3	5	5	6	
Headway (min)	6	6	6	6	
Max. Allowable Headway	7.74	7.53	7.51	9.32	
Slack Time (min)	4.53	0.67	2.22	0.22	
Fleet Size Per Zone (buses)	6	8	6	5	
Operating Cost ($/min)	15.96	36.67	27.50	26.60	106.73
In-Vehicle Cost ($/min)	16.26	80.83	54.56	27.34	178.99
Waiting Cost($/min)	3.12	14.11	12.73	6.95	36.91
Access Cost ($/min)	0.00	41.18	38.10	24.84	104.12
Induced Slack Cost ($/min)	2.16	0.81	3.05	0.20	
Inter-cycle Waiting Cost ($/min)					11.46
Missed Connection Cost ($/min)					4.41
Delayed Connection Cost ($/min)					0.58
Transfer Cost ($/min)					22.67
Total Cost ($/min)					449.42

Case 3: All-Conventional Service Solution: High Demand Case

In Case 2, either conventional or flexible services are chosen, with quintupled the demand of Case 1. In this third numerical example, we increase demand inputs by a factor of 10 compared to Case 1. This case is designed to explore the high demand densities at which conventional services are preferred to flexible services. This case is also designed to have identical line-haul distances and regional characteristics in order to explore whether the common headway solution is preferable to integer multiple headways.

Input Values for Case 3

Table 14 Demand and Other Inputs

Demand (trips/min)					
Region / Period	A	B	C	D	T
A	0.00	1.10	1.40	0.20	0.10
B	2.20	0.00	13.30	13.30	4.40
C	6.90	13.90	0.00	6.90	5.60
D	0.20	5.60	1.10	0.00	0.10
T	0.30	1.10	1.70	0.20	0.00
Region					
Line-haul Distance (miles)	5	5	5	5	
Length of Region (miles)	4	4	4	4	
Width of Region (miles)	3	3	3	3	

Uncoordinated Operation Results in Case 3

With demand inputs increase tenfold, Table 15 shows that all regions are served with conventional services. The vehicle size for region D increases to 25 seats per bus. Other regions have vehicle sizes identical with the results of Case 2. However, their number of zones, fleet size per zone, and headways are adjusted to high demand inputs. The total cost of uncoordinated operations is 689.66 $/ minute.

Table 15 Uncoordinated Operation Results in Case 3

	Regions				sum
	A	B	C	D	
Service Type	1	1	1	1	
Vehicle Size (seats)	16	25	25	25	
Number of Zones	4	6	6	5	
Headway (min)	6.09	4.35	4.35	5.07	
Fleet Size Per Zone (buses)	5	7	7	6	
Operating Cost ($/min)	17.73	38.50	38.50	27.50	122.23
In-Vehicle Cost ($/min)	23.19	102.68	95.01	51.62	272.51
Waiting Cost($/min)	6.33	20.95	18.50	11.78	57.57
Access Cost ($/min)	17.67	57.65	53.34	33.12	161.78
Transfer Cost ($/min)	8.83	27.15	22.81	16.78	75.57
Total Cost ($/min)	73.76	246.92	228.16	140.81	689.66

Timed Transfers Coordination Results in Case 3

The results of timed transfers are shown in Table 16. Headways are optimized at four minutes for all regions, which is the common headway solution. In Case 2, the common headway solution is six minutes while the headway of Case 3 is four minutes, due to increased demand. We also note that for higher demand inputs, the optimized headways are close to their maximum allowable values. With identical line-haul distances and regional characteristics, the solutions for headways and fleet size are identical among regions. However, the resulting optimized vehicle sizes and numbers of zones for each region are found to differ in order to satisfy different demand inputs for each region. The total cost of timed transfers is 673.93 $/minute. Compared to uncoordinated operations, the total cost decreases by 15.73 $/minute (= 2.28 percent reduction). With this case, we find that the provision of timed transfers reduces the total cost for similar line-haul and regional characteristics. However, the cost reduction in Case 3 (with similar round travel times) is lower than in Case 2 (with various round travel times). Due to the similar geographic inputs, conventional services are chosen for all regions in Case 3 whereas the integration of conventional and flexible services is selected in Case 2. This case confirms that the total cost reduction due to service type integration is greater when regions have dissimilar round travel times.

Table 16 Timed Transfers Coordination Results in Case 3

	Regions				sum
	A	B	C	D	
Service Type	1	1	1	1	
Vehicle Size (seats)	16	25	25	25	
Number of Zones	4	6	6	5	
Headway (min)	4	4	4	4	
Max. Allowable Headway	6.67	4.52	4.50	6.07	
Slack Time (min)	1.56	1.56	1.56	1.56	
Fleet Size Per Zone (buses)	8	8	8	8	
Operating Cost ($/min)	28.37	44.00	44.00	36.67	153.04
In-Vehicle Cost ($/min)	23.19	102.68	95.01	51.62	272.51
Waiting Cost($/min)	4.20	19.44	17.03	9.34	50.01
Access Cost ($/min)	17.67	57.65	53.34	33.12	161.78
Induced Slack Cost ($/min)	1.48	3.56	4.03	1.86	
Inter-cycle Waiting Cost ($/min)					24.68
Missed Connection Cost ($/min)					0.84
Delayed Connection Cost ($/min)					0.13
Transfer Cost ($/min)					36.58
Total Cost ($/min)					673.93

Case 4: All-Flexible Service Solution: Low Demand Case

Case 3 confirms that conventional services are desirable when demand densities are higher. In Case 4, we consider low demand inputs so that flexible services can be preferable to conventional services. The demand inputs are 10% of Case 1 inputs. Other parameters have the same values used in Case 1.

Input Values for Case 4

Table 17 Demand and Other Inputs

Demand (trips/min)					
Region / Period	A	B	C	D	T
A	0.00	0.01	0.01	0.00	0.00
B	0.02	0.00	0.13	0.13	0.04
C	0.07	0.14	0.00	0.07	0.06
D	0.00	0.06	0.01	0.00	0.00
T	0.00	0.01	0.02	0.00	0.00

Uncoordinated Operation Results in Case 4

Table 18 shows that all regions are served with flexible services, as expected. The optimized vehicle sizes are also reduced to seven or 10 seats per vehicle. The number of zones is one, for all regions; thus each vehicle round trip covers an entire region. Optimized headways vary between 15 and 30 minutes. Access costs are all zero because flexible services are chosen for all regions. For region A, only one vehicle with seven seats can service all demand. The total cost of uncoordinated operations is 20 $/minute.

Table 18 Uncoordinated Operation Results in Case 4

	Regions				sum
	A	B	C	D	
Service Type	2	2	2	2	
Vehicle Size (seats)	7	10	10	7	
Number of Zones	1	1	1	1	
Headway (min)	30.87	16.03	17.31	15.73	
Fleet Size Per Zone (buses)	1	4	3	3	
Operating Cost ($/min)	0.86	3.47	2.60	2.57	9.49
In-Vehicle Cost ($/min)	0.32	2.93	2.20	1.09	6.54
Waiting Cost($/min)	0.32	0.74	0.73	0.36	2.15
Access Cost ($/min)	0.00	0.00	0.00	0.00	0.00
Transfer Cost ($/min)	0.44	0.95	0.87	0.86	3.12
Total Cost ($/min)	1.94	8.09	6.44	4.53	21.00

Timed Transfers Coordination Results in Case 4

The optimized headways are 32, 12, 12, and 16 minutes for regions A, B, C, and D. Regions B, C, and D are synchronized with the base cycle of two minutes. As shown in Table 17, the demand inputs for region A are lower than for other regions. The lower demand in region A explains the optimized headways. If the demand inputs for region A were considerably higher than the current inputs, headway solutions with the four minutes base cycle may result. The total cost of timed transfers is 20.26$/minute and is thus reduced by 0.74$/minute (3.52%) compared to uncoordinated operations. This case also confirms that timed transfers become increasingly desirable for coordinating passenger transfers among regions that have significantly different travel distances.

Table 19 Timed Transfers Coordination Results in Case 4

	Regions				Sum
	A	B	C	D	
Service Type	2	2	2	2	
Vehicle Size (seats)	7	10	10	7	
Number of Zones	1	1	1	1	

Headway (min)	32	12	12	16	
Max. Allowable Headway	56.45	18.21	19.69	25.36	
Slack Time (min)	0.85	1.30	1.54	0.59	
Fleet Size Per Zone (buses)	1	5	4	3	
Operating Cost ($/min)	0.86	4.33	3.47	2.57	11.23
In-Vehicle Cost ($/min)	0.32	2.69	1.97	1.09	6.06
Waiting Cost($/min)	0.33	0.55	0.51	0.37	1.76
Access Cost ($/min)	0.00	0.00	0.00	0.00	0.00
Induced Slack Cost ($/min)	0.02	0.11	0.13	0.03	
Inter-cycle Waiting Cost ($/min)					0.86
Missed Connection Cost ($/min)					0.05
Delayed Connection Cost ($/min)					0.00
Transfer Cost ($/min)					1.21
Total Cost ($/min)					20.26

CONCLUSIONS

In this paper, uncoordinated and coordinated (timed transfers) bus operations are formulated for both conventional and flexible services. For uncoordinated operations, either analytic methods (for conventional services) or numerical methods (for flexible services) are used to optimize the decision variables, namely the service type, vehicle size, the number of zones per region, headway, and fleet size. A probabilistic optimization model is then proposed for integrating conventional and flexible services with timed transfers. For such coordination of passenger transfers, common headways (or integer multiple headways), their slack times, and the resulting fleets are optimized.

With numerical examples, it is confirmed that the integration of conventional and flexible services is especially desirable when demand densities vary considerably among the served regions. The proposed models quantify the delay and cost savings achievable from optimized coordination. Their results confirm that timed transfers are desirable for increasing the probability of vehicle connections at transfer terminals, and thus minimizing passenger wait times compared to uncoordinated operations. While only a limited number of input combinations can be considered within this paper, the models proposed here can be used to determine when various integration and coordination options are preferable and to quantify their effects on service levels, costs and other measures of effectiveness.

Extensions of this work might consider demand variations among periods as well as regions, the sharing of vehicles and drivers among conventional and flexible services and possible mixing of coordinated and uncoordinated routes (with the latter being favored when optimal headways are small and the variability of bus arrival times is relatively large). Planning models such as those proposed above might also be combined with real-time models for dispatching vehicles from transfer terminals, for controlling and assisting their movements (e.g., through priorities at signalized intersections) and for fully determining flexible routes and schedules (and thus replacing the Stein (1978) approximations for tour lengths). More geographically realistic models, based on Geographic Information Systems (GIS) may also improve the applicability of models for integrating and coordinating public transportation services.

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APPENDIX B

Welfare Maximization for Bus Transit Systems with Timed Transfers and Financial Constraints

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Welfare Maximization for Bus Transit Systems with Timed Transfers and Financial Constraints

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SUMMARY

Conventional and flexible bus services may be combined to better serve regions with a wide range of characteristics. If demand densities and resulting service frequencies are low, the coordination of bus arrivals at transfer stations may significantly reduce passenger transfer times. A method is proposed for integrating, coordinating and optimizing bus services while considering many-to-many travel patterns, demand elasticity, financial constraints and appropriate service type for various regions. The objective is to maximize welfare, i.e., the sum of producer and consumer surplus. The problem is solved with a hybrid optimization method, in which a genetic algorithm with bounded integer variables is selected for solving one of the subproblems. The service types, fares, headways and service zone sizes are jointly optimized. Sensitivity analyses explore how the choice among conventional and flexible buses depends on the demand, subsidy and demand elasticity parameters. The results also show that welfare can increase due to coordination, and these increases are found to be higher in cases with high demand or low subsidy.

KEY WORDS: public transport; hybrid optimization; genetic algorithm; flexible-route bus

1. INTRODUCTION

In general, conventional bus services, with fixed routes and schedules, are preferable at higher demand densities than flexible route services [1]. Combining these different service types may be beneficial when demand densities vary considerably over the service area [2]. With either service type, at low demand densities, the practical headways, and hence wait times, tend to be large. Long headways increase the desirability of coordinating bus arrivals from different routes at transfer stations, in order to reduce passenger wait times at those transfer stations. This paper addresses the problem of designing such a system that integrates conventional and flexible service types while also coordinating vehicle arrivals for many-to-many (M-to-M) demand patterns and elastic travel demand. The objective of maximizing social welfare (i.e. net benefit, the sum of operator profit and consumer surplus), subject to a financial constraint, is pursued by determining the appropriate service type and other design variables for each region.

Many studies on the transit network design problem have sought to improve system efficiency, where the objective may be minimizing generalized cost, maximizing consumers' surplus, maximizing total welfare, etc. [3,4]. System performance of transit networks has been investigated under various circumstances, including fixed and elastic demand [5-9]; conventional and flexible services [1,10-13] or the integration of both services [14,15], under a variety of operational and financial constraints [16-22]. A detailed review of the transit route network design problem is presented by Kepaptsoglou and Karlaftis [4].

The coordination of transit routes has also been studied by several researchers. Abkowitz et al. [23] compared four transfer policies between two routes. Domschke [24] applied heuristics and a branch and bound algorithm to minimize the transfer times of passengers. Lee and Schonfeld [25] optimized slack times for one bus route connecting with a rail line at one terminal. This study was later extended for multiple routes and multiple hubs [26,27], as well as for freight transportation [28]. Knoppers and Muller [29] developed probabilistic models showing how the benefit of coordination depends on the frequency of the connecting routes and synchronization control margins. Muller and Furth [30] showed the effect of transfer scheduling, punctuality control and departure control on the transfer time. Yu et al. [31] proposed a dynamic vehicle dispatching model to minimize the total waiting time of passengers, where a forecasting method based on support vector machines was developed. Ibarra-Rojas and Rios-Solis [32] solved the timetabling problem with a multi-start iterated local search to avoid bus bunching and help passenger transfers.

Recently, Kim and Schonfeld [2] integrated conventional and flexible bus services and optimized the combined system to minimize its total cost. The system was first optimized for uncoordinated operations. Then the headways and fleet size were re-optimized for coordinated operations.

Previous public transportation optimization studies have considered various important aspects of bus transit system design. The main contribution of this paper is in combining the following problem aspects: integration of conventional and flexible services, welfare maximization with demand elasticity, multiple regions, and financial constraints. Specifically, elastic demand, multiple regions and financial constraints are three important characteristics of a public transportation system, corresponding to human behavior, network topology and finance. The joint consideration of the above four aspects significantly extends the realism and value of the analysis. Furthermore, the additional efficiencies obtainable by combining service types and coordinating transfers among routes can be substantial.

In this paper, the problem is modeled with an objective of optimizing service type, headways, fares, and service zone sizes for each region in a multiregional bus transit system. The solution is sought with a hybrid algorithm in which the problem is subdivided into two subproblems, and a genetic algorithm with bounded integer variables is implemented to solve one of the subproblems. A numerical example and analyses of sensitivity to various input parameters are then used to demonstrate the method's capabilities.

2. PROBLEM STATEMENT AND FORMULATIONS

2.1. Bus transit systems

An example of multiregional bus transit system is shown in Figure 1. There are m rectangular regions, each denoted as region k, $1 \le k \le m$, and all connected to a transportation terminal. Assume that the origins and destinations of all trips are either at the terminal or in one of the regions. In each region, one of two alternative bus services is exclusively provided to serve all the passengers in the region: conventional bus service with fixed route and fixed headway (Figure 2) or flexible bus service with flexible route and fixed headway (Figure 3). The m regions are divided into zones served by conventional routes (as in region 1) or flexible routes (as in region 2). It should be noted that, together, the many-to-one services from each region to the terminal provide many-to-many service connecting any OD pair in the system.

2.2. Assumptions

Several simplifying assumptions which closely follow Kim and Schonfeld [2] are listed here:

2.2.1. Assumptions for both conventional and flexible services

(1) Demand is uniformly distributed over space within each region, and sensitive to service quality and price.

(2) Bus size and operating cost are uniform throughout the system for both conventional and flexible services.

(3) Bus load factors are unlimited.

(4) Headway h and scheduled bus arrival time at the terminal are identical for all routes served for a single region.

(5) All buses arrive at the transfer station on the predetermined schedules.

(6) Local speed V includes stopping times.

(7) Layover times are negligible.

(8) External costs and benefits are negligible.

2.2.2. Assumptions for conventional services

(1) The service area is divided into several parallel zones with identical zone areas A. In each zone, a route runs along the middle of the zone.

(2) Bus stop spacing s is given. Passengers are assumed to board and exit at the nearest bus stop to their origin and destination.

(3) A bus round trip consists of: (a) line haul distance J from the terminal to a corner of the entire service area at express speed yV, where y is the ratio of express speed to local speed; (b) an average distance $W/2$ from the corner to the center of the zone at non-stop local speed bV, where b is the ratio of non-stop speed to local speed; (c) distance L at

local speed V; (d) returning to the terminal along the reverse of the above routes.

2.2.3. Assumptions for flexible services

(1) The service area is divided into several zones with identical zone areas A. In each zone, there is one flexible bus route.

(2) Passengers are assumed to board and exit at their doorsteps, thus eliminating access times.

(3) A bus round trip consists of: (a) line haul distance J at express speed yV; (b) an average distance $\left(L+W\right)/2$ from the corner to the center of the service zone at non-stop local speed bV; (c) a local collection and distribution tour with length l (which will be formulated later) at local speed V; (d) returning to the terminal along the reverse of (a)&(b).

2.3. Notation

Definitions, units and baseline values of variables and parameters are listed in Table I.

2.4. Demand function

q^{ij} denotes the potential demand for travel from region i to region j by bus, $0 \le i, j \le m$. When $i=0$ or $j=0$, the origin or the destination is the terminal. Then the demand function is

$$Q^{ij} = \max\left\{q^{ij}\left[1-e_w t_w^i - e_x\left(t_x^i + t_x^j\right) - e_v\left(t_v^i + t_v^j\right) - e_w t_t^{ij} - e_p\left(f^i + f^j\right)\right], 0\right\} \qquad (1)$$

where Q^{ij} is the actual demand from region i to region j; t_w^i, t_x^i and t_v^i are the average user wait, access and in-vehicle time in region i; t_t^{ij} is the average user transfer time from region i to region j at the terminal; f^i is the fare in region i; e_w, e_x, e_v and e_p are the demand elasticity parameters for wait time, access time, in-vehicle time and fare respectively. $t_w^0 = t_x^0 = t_v^0 = t_t^{i0} = f^0 = 0$. $t_t^{0j} = z_1 h^j$, where h^j is the headway of buses serving region j.

Note that the concept of Equation (1) is adapted from Zhou et al. [20], where the demand between a single region and a terminal is formulated. To integrate conventional and flexible services in multiple regions in this study, denote x^k as the service type of region k, where $x^k = 1$ for conventional service and $x^k = 0$ for flexible service. Then, the average user in-vehicle time is expressed as

$$t_v^k = x^k t_{v,c}^k + \left(1 - x^k\right) t_{v,f}^k \tag{2}$$

where $t_{v,c}^k$ is the average user in-vehicle time in region k for conventional services, $t_{v,f}^k$ is the average user in-vehicle time in region k for flexible services. However, the average user access time $t_x^k = x^k t_{x,c}^k$ since user access times are eliminated for door-to-door flexible services. The average user wait time $t_w^k = 0.5 h^k$ is similar for both service types. The average wait time parameter 0.5 is reasonable here since the headways are assumed to be equal and buses are assumed to arrive on-time. The formulations of $t_{v,c}^k$ and $t_{x,c}^k$ are similar to those introduced by Zhou et al. [20].

The average user in-vehicle time for flexible services is given by

$$t_{v,f}^k = \frac{J^k}{y^k V^k} + \frac{L^k}{2b^k V^k} + \frac{W^k}{2b^k V^k} + \frac{l^k}{2V^k} \tag{3}$$

where l^k is the collection tour distance estimated with Stein's formula [33]: $l^k = \phi\sqrt{n^k A^k} = \phi A^k \sqrt{q^k h^k / u^k L^k W^k}$, in which $q^k = \sum_{j=0}^{m} q^{kj} + \sum_{i=0}^{m} q^{ik}$ and $\phi = 1.15$ for rectilinear spaces [34]. Note that Equation (3) differs from Zhou et al. [20] due to assumption 3 for flexible services applied in this study.

The detailed formulation of t_t is presented in the next section.

2.5. Average transfer time of uncoordinated and coordinated transit systems

If the buses are uncoordinated, i.e., the schedules of buses for different regions are unrelated, the passengers arrive at the terminal (on buses) randomly with respect to the scheduled departure time of other buses. Then, the average transfer time is $t_t^{ij} = 0.5h^j$, $i, j \neq 0$.

If bus arrivals are coordinated, the transfer time may be reduced. In formulating this case, assume that the first buses from different regions arrive simultaneously at the terminal. Then, a periodic synchronization will be achieved for the later buses. Figure 4 shows an example of coordination. There are two regions with bus services. The headways of the buses in regions 1 and 2 are 6 minutes and 10 minutes, respectively. Every 30 minutes, the buses of the two regions meet at the terminal. The average user transfer times are: $t_t^{12} = (0+4+8+2+6)/5 = 4, t_t^{21} = (0+2+4)/3 = 2$.

Generally, given headways for region i and j of h^i and h^j, the average transfer time t_t^{ij} can be calculated as follows: first, define $d = \text{GCD}(h^i, h^j)$ when h^i and h^j are integers, where GCD stands for greatest common divisor. If h^i or h^j is not integer but a rational number, express h^i and h^j as $h^i = a^i / p$ and $h^j = a^j / p$, where a^i, a^j and p are integers. Then, define $d = \text{GCD}(a^i, a^j) / p$. The average transfer time is calculated as:

$$t_t^{ij} = \frac{d}{h^j} \sum_{k=0}^{\frac{h^j}{d}-1} (kd) = \frac{d}{h^j} \left[\frac{1}{2} \times \left(\frac{h^j}{d} - 1 \right) d \times \frac{h^j}{d} \right] = \frac{h^j}{2} - \frac{d}{2} \tag{4}$$

where $i, j \neq 0$.

From Equation (4), it can be observed that t_t^{ij} decreases as d increases, which indicates the relation between the form of headways and the transfer time under coordinated bus transit systems: as GCD increases, transfer time decreases. If headways are fairly uniform but unrelated among routes, t_t^{ij} is close to $0.5h^j$, and the bus routes are nearly uncoordinated; if the headways are identical, transfer delay time approaches zero. Similar results can be found in Ting [35] and Ting and Schonfeld [27].

2.6. Financial performance measures

Given the passengers' demand, the revenue of the entire bus system (without subsidy) is

$$R = \sum_{k=1}^{m} f^k \left(\sum_{j=0}^{m} Q^{kj} + \sum_{i=0}^{m} Q^{ik} \right) \tag{5}$$

Then the producer surplus, more commonly called profit (generally nonpositive in this problem), is $P = R - C$, where C is the operator cost and can be calculated from the headway and bus round trip time.

The consumer surplus, which is the difference between the consumer's willingness to pay and the amount they actually pay for the services, is $G = \sum_{i=0}^{m} \sum_{j=0}^{m} G^{ij}$, where

$$G^{ij} = \begin{cases} \dfrac{\left(Q^{ij}\right)^2}{2e_p q^{ij}} & \text{if } q^{ij} > 0 \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

and this expression is explained in Kocur and Hendrickson [8].

Total Welfare (i.e. net benefit), which is the sum of the consumer surplus and the producer surplus, is then expressed as $Y = G + P$.

2.7. Optimization model

The objective of this study is to maximize the total welfare in the system, including that of the transit operator and its customers. Because the increased consumer surplus due to serving additional passengers is included in the welfare but ignored in the total cost, the total welfare is preferable to the total cost in expressing the net value of providing services. Assume that a system-wide subsidy applies to all services and regions. The problem is formulated as the following optimization problem:

$$\max \ Y \tag{7}$$

s.t.

$$P + S_{\max} \geq 0 \tag{8}$$

where $S_{\max}$ denotes the predetermined total maximum allowable subsidy for the transit systems in all m regions. The constraint guarantees that the losses of operating bus services can be fully compensated, but does not guarantee positive profits.

The decision variables include: x^k, which denotes the service type of buses in region k, either conventional or flexible; fare f^k, headway h^k, and each route's service zone area A^k in region k.

3. OPTIMIZATION METHOD AND NUMERICAL RESULTS

The optimization problem (7)–(8) is a nonconvex mixed-integer nonlinear program. The welfare function is a discrete step function with respect to the service type in a region. The GCD (greatest common divisor) function introduces further steps in the objective function. Therefore, convex optimization methods will not guarantee an optimal solution. A random-restart interior point method was developed, but was found to be inefficient. A metaheuristic, specifically a hybrid genetic algorithm (GA) is proposed as an alternative. A GA tests a series of potential solutions (represented by chromosomes) through the process of reproduction and recombination (see Whitley [36] for a tutorial to GAs). As there are only a few possible combinations of service types (i.e. fixed or flexible route), these combinations are completely enumerated. Thus, the chromosomes of the proposed hybrid GA contain only continuous decision variables, namely fare (f^k), headway (h^k) and service zone area (A^k) of each region k.

The original optimization problem can be reformulated as follows:

$$\max_{\{x,f,h,A\}} \quad Y\left(x,\ f,h,A\right) \tag{9}$$

s.t.

$$\left\{f,h,A\right\} = \underset{\{f',h',A'\}}{\arg\max}\left\{Y\left(x,f',h',A'\right): P+S_{\max} \geq 0\right\} \tag{10}$$

In this model, the variables are separated into two categories that are optimized sequentially by dividing the problem into two subproblems:

Subproblem 1: Choose the combinations of the service types (x^k) of each region k.

Subproblem 2: Choose fare (f^k), headway (h^k) and service zone area (A^k) of each region k.

When the number of regions m is relatively low (e.g., $m \leq 5$), subproblem 1 can be solved by enumerating all 2^m possibilities. When m is large, a heuristic method may be desirable.

For subproblem 2, a genetic algorithm with bounded integer variables is implemented. Fare (f^k) and headway (h^k) are set to integers or to multiples of a chosen basis. In addition, the number of routes in each region is required to be integer, which leads to several discrete available service zone areas (A^k) for each region k. Then the feasible region becomes a discrete subset of the original feasible region of subproblem 2. Although the original optimal solution may not be achieved under integer restrictions, the benefits of such changes are that: (*a*) in practice, these values are usually discrete and bounded and (*b*) the chromosomes are short, increasing the algorithm's efficiency.

Then, in the algorithm, the chromosome is a sequence of f, h and the number of routes in each region. The fitness function is $-Y$. The algorithm maintains a fixed population size in each generation, some of whose members are elite offspring (individuals that are guaranteed to survive from the previous generation). 80% of the remaining children are generated through crossover and the others are generated through mutation. A Laplace crossover and a power mutation proposed by Deep et al. [37] are applied. A GA with such crossover and mutation functions, which is named a real coded genetic algorithm, is designed for solving integer and mixed integer constrained optimization problem by incorporating a special truncation procedure.

The algorithm is applied to examine a numerical example with 3 regions. The length, width and line haul distance of each region are listed in Table II. The potential demands are listed in Table III. The maximum allowable subsidy $S_{\max} = 600$ \$/hour. Other parameters are set to their baseline values, as shown in Table I. To solve the sample problem, all possible combinations of service types are enumerated in subproblem 1, while in subproblem 2, the parameters of the genetic algorithm are set as: a population size of 500, 50 elite offspring in each generation, and up to 2000 generations. The solution of each subproblem 2 is obtained after 10 replications of the genetic algorithm.

In the GA for this example, Fare (f^k) is a multiple of $0.1, whose upper bound is $10; headway (h^k) is in integer minutes with an upper bound of 1 hour. In Table IV, the optimized solutions of the uncoordinated and coordinated bus systems are compared. It can be observed that the welfare of the coordinated system exceeds that of the uncoordinated system by $\left(5947-5445\right)/5445\times100\%=9.22\%$, which can be regarded as the relative benefit of coordination. The optimized headways in the coordinated system for all three regions are the same, which means that the arrival times at the terminal of all buses can be fully synchronized, and all the transfer times are 0. This is an important advantage for increasing the demand and welfare, compared to the uncoordinated system.

Next, the benefits of integrated service types are shown in Table V. The optimized system welfare under uniform service types (i.e., the bus service types in the 3 regions are either all conventional or all flexible) are compared with the results for the integrated service types. It is shown that in this numerical example, compared to the uniform service types, the integration of conventional and flexible service types, whether coordinated or uncoordinated, can increase the welfare by 0.49–2.14%. Note that other inputs may lead to higher benefits from the integration of service types. For example, if the demand between regions 1 and 3 increases by a factor of 5, integration improves welfare by 5.62%.

4. SENSITIVITY ANALYSES

In this section, how small changes in the input variables and parameters affect the results are analyzed. Based on the numerical example in the last section, the sensitivity analyses are conducted for: (*a*) potential demand of the bus services q; (*b*) maximum allowable subsidy $S_{\max}$; and (*c*) demand elasticity parameter for access time e_x.

4.1. Sensitivity to potential demand of the bus services

To analyze the effect of small changes in demand, a series of demand multipliers are set. The new potential demand is then the original potential demand listed in Table III multiplied by a demand multiplier. The main contents of Table VI show the optimized coordinated system, while the last three lines show the welfare of the optimized uncoordinated system, the benefit, and the relative benefit of coordination with each demand multiplier.

It can be observed that as the demand multiplier grows from 1 to 4, the optimized service types remain unchanged, while both headways and service zone areas decrease so that the system can serve more passengers. As a result, the welfare, as well as the benefit of coordination, grows rapidly. However, the relative benefit of coordination decreases since, as demand grows, the optimized uncoordinated headways and resulting transfer times also decrease.

When the demand multiplier decreases from 1 to 0.25, the optimized service types in some regions becomes flexible instead of conventional. Such results numerically verify the previous findings that conventional service is suitable for high demand and flexible service is suitable for low demand (Chang and Schonfeld 1991a), although the measure here is welfare rather than costs, which are used in most other studies.

4.2. Sensitivity to maximum allowable subsidy

Next, the effects of various subsidy levels are analyzed. The results are shown in Table VII. As the allowable subsidy increases, the welfare grows slowly. Here conventional buses operate in 2 regions under all subsidy levels and undertake most of the passenger trips. Such behavior has been observed by Chang and Schonfeld [16] and by Zhou et al. [20] for single region bus transit systems: the welfare vs. subsidy curve is relatively flat for conventional services and steep for flexible services.

It is also notable that as the allowable subsidy increases, both the absolute and relative benefits of coordination decrease. This is explainable as follows: as the subsidy increases, there is more scope for improving the transit system. Specifically, the fare and headways can be reduced to improve the welfare. Then, the benefit of coordination decreases with decreasing headways while the change in actual demand is insignificant.

Now we check the producer surplus, $\sum_{k=1}^{m} P^k$ for each subsidy level. It can be observed that for $S_{\max} = 0, 300, 600, \text{or } 900$, the constraints $\sum_{k=1}^{m} P^k + S_{\max} \geq 0$ are nearly binding. For $S_{\max} = \infty$, the fare is 0 in each region. Under the settings in this problem, the objective of maximizing total welfare encourages the transit agency to set the fare as low as possible if the loss can be offset by the subsidy. Such a result is consistent with the role of nonprofit transit agencies.

4.3. Sensitivity to demand elasticity parameter for access time

Finally, one of the demand elasticity parameters, e_x, which represents passengers' sensitivity to the access time, is adjusted. The results are shown in Table VIII. As e_x increases, the flexible services become increasingly preferable, since there is no access time for flexible services. Also, according to the demand function, when e_x increases the number of travelers using conventional buses decreases, which decreases welfare. It can be observed that the benefit of coordination increases as e_x increases until $e_x = 0.5$. When $e_x = 0.5$, the resulting service types in all regions become flexible. Increasing e_x further will not affect the optimal operating strategy.

5.CONCLUSIONS

A welfare maximization problem for bus transit systems with many-to-many demand patterns and elastic travel demand is analyzed here. Two types of bus services, namely conventional and flexible services, are integrated for a multiple region system. The arrivals of buses serving different regions are coordinated at a transfer station to reduce passenger transfer times. The problem is formulated as an optimization problem and solved with a hybrid algorithm in which an efficient genetic algorithm with bounded integer variables is selected for solving one of the subproblems. The service type, headway, fare and service zone area of each region are jointly optimized subject to a systemwide subsidy constraint.

Sensitivity analyses yield the following findings. The conventional services are increasingly preferable at high demand and low values of the access time elasticity parameter e_x. Flexible services are preferable at low demand and high e_x. No significant effects of maximum allowable subsidy on the optimal service types are observed. To precisely determine the most suitable service types, detailed input data are still needed to optimize the service type for each region under various conditions. For all tested input parameters, the optimized headways in the coordinated system for all three regions are the same, so that all bus routes are fully synchronized to eliminate the transfer delays. Therefore, for coordinated systems, if a full search of possible headway combinations (like the heuristics in this study) is considered too costly, the headway search can be focused on: (a) common headways, as previously done by Chien [38] or

(b) integer-ratio headways, as previously done by Ting [35] and by Ting and Schonfeld [27]. It should be noted that all the possible headways searched in this study are integer minutes, which can also be regarded as integer-ratio headways with a short (1 minute) base cycle. Compared to uncoordinated operations, coordinated operations were found to enhance welfare by 5%–20%, depending on circumstances. Coordination increases welfare at high demand and low subsidy. However, the relative benefits of coordination are higher at low demand.

The analyses show the interrelations among various parameters and the system performance measures for relatively complex transit systems with elastic demand, multiple service types, timed transfers, and financial constraints. Such results can support the efficient planning and design of realistic transit systems.

Finally, some possible extensions of this study include the following. The linear demand functions assumed in this study may be replaced by nonlinear functions which might better fit observed relations in some environments. Changing the structure of the demand function may lead to different results on the relative merits of service types and other decision variables under optimization. Additionally, the bus arrival times are assumed to be deterministic in the analysis. If the arrival times are treated as stochastic, optimized slack times (i.e. differences between the expected arrival times and the scheduled departure times) should be considered in estimating transfer times. Moreover, the increasing uncertainty of transfer times may decrease the attractiveness of transfer routes, and hence the demand [39]. Finally, the demand may also be treated as probabilistic variables, which would then influence dwell times at stops and arrival distributions at successive stops.

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Table I. Variable and parameter definitions.

Symbol	Unit	Description	Baseline value
A	mi^2	Zone area	—
a		Numerator of headway h	—
b		Non-stop ratio = non-stop speed/local speed	2
C	$/h	Operator cost	—
D	h	Bus round trip time	—
e_p		Demand elasticity parameter for fare	0.03
e_v		Demand elasticity parameter for in-vehicle time	0.15
e_w		Demand elasticity parameter for wait time	0.3
e_x		Demand elasticity parameter for access time	0.45
f	$/trip	Fare	—
G	$/h	Consumer surplus	—
h	h	Headway	—
J	mi	Line haul distance of a region	—
L	mi	Length of a region	—
l	mi	Collection tour distance	—
m		Number of regions	—
n		Number of pickup stops	—
P	$/h	Producer surplus (profit)	—
p		Denominator of headway h	—
Q	trips/h	Actual demand of the bus services	—
q	trips/h	Potential demand of the bus services	—
R	$/h	Revenue	—
S	$/h	Subsidy	—
t_t	h	Average user transfer time	—
t_v	h	Average user in-vehicle time	—
t_w	h	Average user wait time	—
t_x	h	Average user access time	—
u		Average number of passengers per pickup point	1.2
V	mi/h	Local speed	20
W	mi	Width of a region	—

x		Service type (1: conventional, 0: flexible)	—
Y	$/h	Social welfare	—
y		Express ratio = express speed/local speed	2
ϕ		Constant in the collection distance equation	1.15

— = not applicable.

Table II. An example of multiregional bus transit system.

Region	Length (mi)	Width (mi)	Line haul distance (mi)
1	4	5	10
2	3	2	5
3	6	5	4

Table III. Potential demand for bus services (trips/h).

	Destination region			
Origin region	Terminal	1	2	3
Terminal	0	80	20	100
1	40	0	30	130
2	20	30	0	50
3	80	100	40	0

Table IV. Numerical results for the uncoordinated and coordinated bus systems.

Variables	Coordinated			Uncoordinated		
Region	1	2	3	1	2	3
Service type	Conv.	Flex.	Conv.	Conv.	Flex.	Conv.
Fare ($/trip)	1.2	0.6	0.7	1	1.1	1.2
Headway (min)	26	26	26	23	15	20
Service zone area (mi^2)	5	2	6	5	3	7.5
Actual Demand (trips/h)	282	142	361	267	134	338
Operator cost ($/h)	473	264	534	535	327	555
Revenue ($/h)	339	85	252	267	147	406
Producer surplus ($/h)	-134	-179	-281	-268	-180	-149
Welfare of the coordinated system ($/h)				5947		
Welfare of the uncoordinated system ($/h)				5445		

Table V. Results comparison between integrated and uniform service types.

	Coordinated			Uncoordinated		
Service types	Integ.	Conv.	Flex.	Integ.	Conv.	Flex.
Welfare ($/h)	5947	5918	5842	5445	5411	5331
Relative benefit bet. integ. and alternatives		0.49%	1.80%		0.63%	2.14%

Table VI. Sensitivity of coordinated bus systems to demand multiplier.

	Demand multiplier				
Variables	0.25	0.5	1	2	4
Region 1					
Service type	Flex.	Flex.	Conv.	Conv.	Conv.
Fare ($/trip)	0.9	0.8	1.2	0.9	0.8
Headway (min)	32	34	26	22	16
Service zone area (mi^2)	10	5	5	3.333	2.857
Region 2					
Service type	Flex.	Flex.	Flex.	Flex.	Flex.
Fare ($/trip)	0.6	0.5	0.6	0.8	0.7
Headway (min)	32	34	26	22	16
Service zone area (mi^2)	6	3	2	1.2	1
Region 3					
Service type	Flex.	Conv.	Conv.	Conv.	Conv.
Fare ($/trip)	1.5	1.2	0.7	1.1	1.0
Headway (min)	32	34	26	22	16
Service zone area (mi^2)	7.5	7.5	6	4.286	3.75
Producer surplus ($/h)	-575	-599	-594	-595	-544
Welfare of the coordinated system ($/h)	1015	2522	5947	13408	29327
Welfare of the uncoordinated system ($/h)	857	2222	5445	12578	27951
Benefit of coordination ($/h)	158	300	502	830	1376
Relative benefit of coordination	18.44%	13.50%	9.22%	6.60%	4.92%

Table VII. Sensitivity of coordinated bus systems to maximum subsidy.

	Maximum allowable subsidy ($/h)				
Variables	0	300	600	900	∞
Region 1					
Service type	Conv.	Conv.	Conv.	Conv.	Conv.
Fare ($/trip)	1.5	1.5	1.2	0.3	0
Headway (min)	27	28	26	25	25
Service zone area (mi^2)	5	5	5	5	5
Region 2					
Service type	Flex.	Flex.	Flex.	Flex.	Flex.
Fare ($/trip)	0.8	0.7	0.6	0.5	0
Headway (min)	27	28	26	25	25
Service zone area (mi^2)	2	2	2	2	2
Region 3					
Service type	Conv.	Conv.	Conv.	Conv.	Conv.
Fare ($/trip)	2.3	1.1	0.7	0.8	0
Headway (min)	27	28	26	25	25
Service zone area (mi^2)	6	6	6	6	6
Producer surplus ($/h)	46	-293	-594	-865	-1319
Welfare of the coordinated system ($/h)	5877	5917	5947	5960	5970
Welfare of the uncoordinated system ($/h)	5361	5411	5445	5466	5489
Benefit of coordination ($/h)	516	506	502	494	481
Relative benefit of coordination	9.63%	9.35%	9.22%	9.04%	8.76%

Table VIII. Sensitivity of coordinated bus systems to access time elasticity parameter.

	Access time elasticity parameter				
Variables	0.30	0.40	0.45	0.5	0.6
Region 1					
Service type	Conv.	Conv.	Conv.	Flex.	Flex.
Fare ($/trip)	0.2	0.4	1.2	1.8	1.8
Headway (min)	23	27	26	24	24
Service zone area (mi^2)	6.667	5	5	4	4
Region 2					
Service type	Conv.	Conv.	Flex.	Flex.	Flex.
Fare ($/trip)	0.9	0.7	0.6	1.6	1.6
Headway (min)	23	27	26	24	24
Service zone area (mi^2)	3	3	2	2	2
Region 3					
Service type	Conv.	Conv.	Conv.	Flex.	Flex.
Fare ($/trip)	0.7	0.8	0.7	2.2	2.2
Headway (min)	23	27	26	24	24
Service zone area (mi^2)	7.5	6	6	3.75	3.75
Producer surplus ($/h)	-587	-595	-594	-580	-580
Welfare of the coordinated system ($/h)	6509	6104	5947	5840	5840
Welfare of the uncoordinated system ($/h)	6036	5611	5445	5334	5334
Benefit of coordination ($/h)	473	493	502	506	506
Relative benefit of coordination	7.84%	8.79%	9.22%	9.49%	9.49%

APPENDIX C

Development of a mode choice model for general purpose flexible route transit systems

This article is in preparation for future journal publication.

DEVELOPMENT OF A MODE CHOICE MODEL FOR GENERAL PURPOSE FLEXIBLE ROUTE TRANSIT SYSTEMS

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ABSTRACT
This paper develops a mode choice model that can be used to unveil how transit users select between competing transit options. Specifically, the model choice model considers traditional fixed-route transit systems, flexible-route systems in which vehicles are shared but routes are flexible to prevailing demands and individual transit systems (e.g., Uber or Lyft) that provide door-to-door and demand responsive service. A stated preference survey was performed in which survey participants were provided a set of scenarios and asked to select the most attractive transit option of the three previously mentioned. Each scenario was presented using the following attributes: walking time required, waiting time (including variability), in-vehicle travel time (including variability), monetary cost and availability of GPS tracking services. Various statistical modeling frameworks were considered and applied to these survey data to describe the mode choice decision-making process. The results revealed that some individuals always select the same mode, regardless of the parameters. However, costs, expected in-vehicle waiting time, expected waiting time and walking time were found to be statistically significant predictors of the type of transit option selected.

INTRODUCTION

Collective transportation options are growing in both flexibility and complexity. While the predominant option today is still fixed-route public transportation services, other options are becoming more viable to replace private automobile travel. For example, most transit systems offer shared demand-responsive door-to-door service for a variety of customers. This type of service is typically reserved for those with special needs (as prescribed in the Americans with Disabilities Act). However, various studies have shown that demand-responsive systems might be able to provide better service than traditional fixed-route services with lower cost under certain conditions [1], [2], [3], [4]. Further, the rapid rise of services such as Uber or Lyft has created even more collective transportation options. These shared-vehicle services offer door-to-door transportation on demand while allowing vehicles to be reused for multiple trips.

Future transportation systems should provide a careful balance between fixed and flexible transit options to ensure that the system provides a mix of services that meets all travelers' needs. Unfortunately, transit researchers and practitioners do not currently have a good idea how transit users actually value each of these different types of collective transportation modes and how users will select between them. Existing transit demand models primarily focus on how users select between fixed-route transit and private automobile options (e.g., [5], [6], [7]). These models predict the mode share based on total travel time or cost on each of these modes as explanatory variables. These results cannot provide insight into flexible transit options since they generally do not discern the pertinent features of each. Specifically, flexible transit options offer different combinations of component travel times compared to fixed-route transit options: e.g., walking, waiting, and in-vehicle travel times. Failure to account for how travelers value these different components will lead to inaccurate demand estimates. A few demand models that consider fixed-route transit services (e.g., [8], [9], [10], [11]) break down travel times into their component pieces to account for the fact that travels value these components differently. Others ([8], [12]) also include access distance as an explanatory variable, which can account for the different walking distances for the transit services. However, these are not applicable to flexible transit options since these modes were generally not considered in the model building process; therefore, general preference for these different modes cannot be considered.

Unfortunately, there appears to be no comprehensive model that can help transit planners determine mode choice between fixed and flexible transit options. Thus, the objective of this paper is to examine how transit users might select between three different transit modes: traditional fixed-route transit service, flexible (i.e., demand responsive) transit service and individual transit service (e.g., like Uber or Lyft). A stated-preference survey is used where participants must select between one of these three modes based on the pertinent features of the trip. Three different modeling frameworks were then applied to describe this data. The modeling results were generally consistent across frameworks, which is promising. The three frameworks were compared based on their ability to predict the mode choices and the best framework was identified. The results of this work can be used as a first step to develop more comprehensive demand models that account for both fixed and flexible transit options.

The remainder of this paper is organized as follows. The following section describes the survey instrument and summarizes the data. Next, three logit models for predicting mode choice based on time and monetary variables of the modes as well as socio-economic characteristics of the respondents are presented. The concluding section summarizes the findings of the paper and provides recommendations for future work.

DATA DESCRIPTION

A survey instrument was created to help unveil the desired information. The survey was designed to be short (answered within 10 minutes) and consisted of two sections. The first was a set of scenarios in which participants were presented with hypothetical trips using the three modes and asked to select the most attractive option. This made up the bulk of the survey. The second section included a set of questions on the participant's previous transit experiences/thoughts, as well as basic demographics. The remainder of this section will provide more details on these two sections.

Survey Design

Scenarios

The scenario section provided each user with hypothetical work trip scenarios and three potential options that could be used to make each trip: a fixed-route transit option, a flexible-route transit option and an individual transit option. Participants were then asked to select the most attractive option from the set of three. Various operational attributes were used to describe each of these three hypothetical options. These were:

- **Walking time:** average time required to walk from the origin to transit stop and from the transit stop to destination. This factor is primarily applicable to fixed-route transit systems, although flexible-route systems might have some walking components.
- **Waiting time:** average time spent waiting at the origin or transit station for a vehicle to arrive. For flexible-route or flexible individual, this refers to the time it takes for the vehicle to arrive after you call for it.
- **Waiting window:** window around the average waiting time during which the transit vehicle might arrive. Smaller waiting windows represent more reliable estimates of waiting time.
- **In-vehicle travel time:** average time spent in transit vehicle moving towards the destination.
- **In-vehicle travel time window:** window around the average in-vehicle travel time that the vehicle might take to arrive to the destination. Smaller waiting windows represent more reliable estimates of in-vehicle travel time.
- **Cost:** monetary cost of the trip.
- **GPS:** availability of GPS device that can provide vehicle location to transit users.

TABLE 1 provides the complete set of values that were considered in this analysis for each of the parameters described. A complete factorial design of these potential values was unfeasible as it would result in over 12,500,000,000 unique scenarios to test. Instead, a subset of scenarios was selected for inclusion in the survey using the *optex* procedure in SAS, implementing the sequential search option. An efficient design was selected that consisted of a set of 36 scenario blocks each containing 10 scenarios. In the survey, every participant was randomly presented with one of these scenario blocks and asked to select the most preferred transit option for the 10 scenarios contained within that block.

TABLE 20 Set of parameters considered for suvey scenarios

	Fixed-route	Flexible-route	Individual
Walking time	5, 8, 12, 20 min	0, 5 min	0 min
Waiting time	8, 12, 16 min	5, 15, 30, 60 min	5, 10, 20, 30 min
Waiting window	+/- 2, 5, 8 min	+ 10, 20, 30 min	+ 5, 10, 15 min
In-vehicle travel time (IVTT)	20, 30, 45, 60 min	FIXED +5, +10, -5, -10, 0 min	FIXED - 5, -10
IVTT window	+/- 0, 5, 10 min	+/- 5, 10 min	+/- 0, 5 min
Cost	$0.5, 1.5, 2, 3	100%, 200%, 300%, 400%, 500% of Fixed	500%, 600%, 700%, 800%, 900% of Fixed
GPS	Yes, No	Yes, No	Yes, No

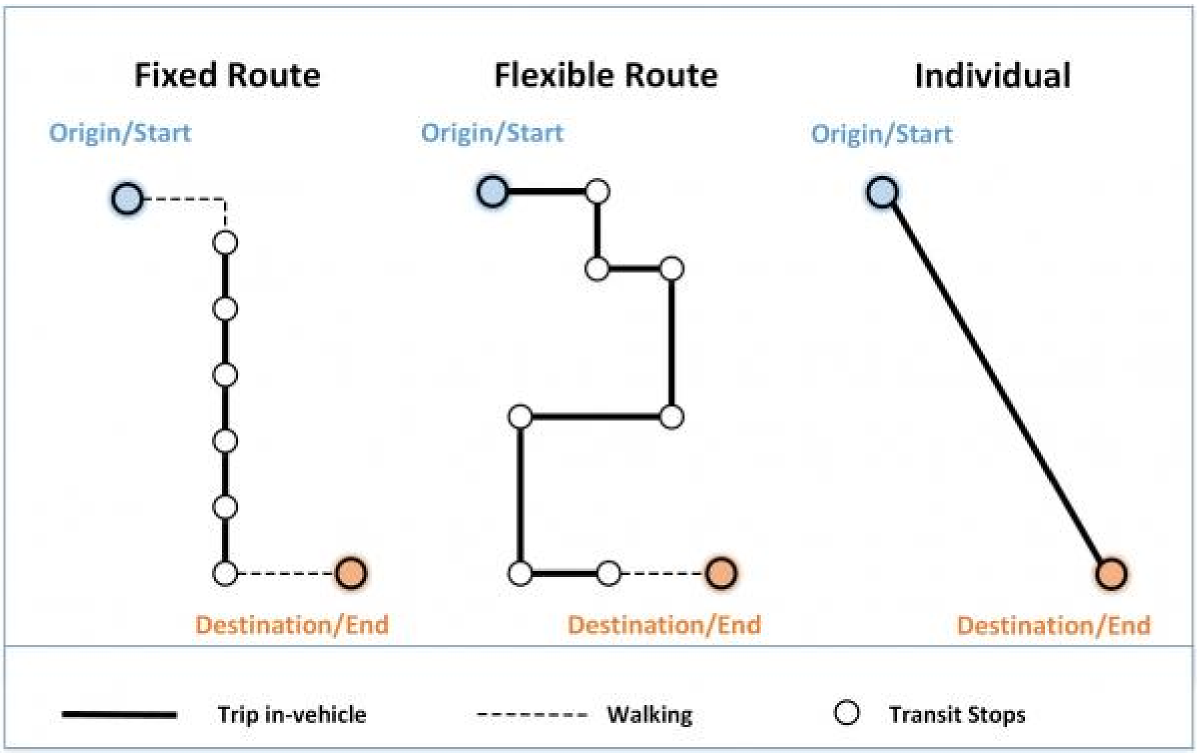


FIGURE 6 Graphical depiction of the three different transit options available in the survey scenarios.

To help survey participants understand some of the qualitative differences between the three transit modes, each participant was provided with a basic description of the pertinent features of each. The exact description used was:

- **Fixed-route** refers to a traditional bus that follows a set or fixed-route. Users typically must walk to a stop to access the transit vehicle, and walk from a stop to their final destination. All waiting for the bus will occur at the bus stop location.
- **Flexible-route** refers to a van or bus with a route that deviates from a fixed-route. This option is similar to a Super Shuttle van. Users are picked up (dropped off) at their origin (destination), or must walk a short distance to (from) a pick up (drop off) location. All waiting for the bus will occur very near the home or the origin location.
- **Individual** refers to a taxi or similar service where one person (or a group) is driven directly from their origin to a destination with no intermediate stops. This option is similar to a traditional taxi. Users are picked up from their origin and are dropped off directly at their destination without having to walk to access the transit vehicle. All waiting for the bus will occur at the home or origin location.

In addition, the graphic presented in Figure 1 was used to illustrate each of these transit options to survey participants. We found the figure to be vital as preliminary survey runs suggested that some users were not familiar with all of these options.

To simplify the presentation of the scenarios, the full set of parameters in Table 1 was reduced when presented to the survey participants. Only the following categories (and accompanying descriptions) were provided:

- **Walking time:** time spent walking from origin to stop or from stop to destination.
- **Waiting time:** time spent waiting for the vehicle to arrive. For fixed-route, this refers to the time spent waiting once you arrive to the stop. For the flexible-route or flexible individual, this refers to the time it takes for the vehicle to arrive after you call for it. [presented as a range that incorporated the window]
- **In-vehicle travel time:** time spent in transit vehicle moving towards your destination. [presented as a range that incorporated the window]
- **Cost**
- **GPS**

TABLE 2 provides an illustration of an example scenario that was provided to the survey participants.

TABLE 21. Example scenario included in survey

Mode	Fixed-route	Flexible-route	Flexible Individual
Walking time	10 min	5 min	-
Waiting time	4-20 min	60-70 min	20-25 min
In-vehicle travel time	35-55 min	45-55 min	40 min
Cost	$ 3	$ 6	$ 18
GPS	No	No	No

Transit experiences and demographics

The transit experiences and demographics section contained several questions about participant's previous experiences in transit and basic demographics. For transit experiences, several statements are included and participants were asked to respond with their agreement level using a 5-point Likert scale. The lowest level is "Strongly disagree" while the highest level is "Strongly agree". These statements were:

- I feel that public transit is safe
- I am comfortable with mobile technology
- I prefer public transit over driving
- Navigating bus systems is easy
- Navigating train system is easy
- I use a smartphone to plan transit trips
- Buses are usually on time
- Taxis are expensive
- I am familiar with smartphone enabled trips such as Uber, Sidecar or Lyft
- I would take the bus more if the stops were closer to my origin and/or destination

In addition, participants were asked to indicate the modes of travel used the past month and the mode used most often. These questions were used to see if previous experienced significantly impacted the type of transit mode selected in the first section of the survey.

Demographic information was also collected about the following characteristics:

- Gender
- Age
- Education
- Income
- Employment status
- Mobility limitations

Data Description

Data was collected via online survey using Qualtrics software, which allows for easy dissemination via an online interface, and in person through a paper survey over 6 months in 2015. Paper surveys were disseminated throughout Baltimore, MD. Key locations included a major transit hub, Mondawmin, and downtown at the inner harbor. The online survey was broadly disseminated to a wide audience using a variety of methods. This included: email listserves of transportation organizations (including the Transportation Research Board's Paratransit Committee (AP060) and through social media.

Overall, a total of 185 of survey responses were received. However, a significant portion of these were incomplete—either missing complete or partial information. After removing these partial responses, we were left with a total of 177 completed surveys used in the analysis. A summary of the socioeconomic characteristics of the respondents is provided in TABLE 3. The sample, which is predominately male (63%), contains a broad cross-section of household income levels. The majority of the respondents are under the age of 35 with nearly half working full-time.

TABLE 22 Socioeconomic characteristics

Gender	**173**	**Household Income**	**153**
Female	37%	<$20,000	10%
Male	63%	$20,000-30,000	14%
Age	**173**	$30,000-50,000	22%
<18	2%	$50,000-75,000	23%
18-25	29%	$75,000-100,000	17%
25-35	31%	>$100,000	14%
36-45	10%	**Employment Status**	**164**
46-55	13%	Retired/Don’t work	6%
56-65	12%	Unemployed, looking	9%
>65	3%	Self-employed	10%
Education Level	**172**	Employed, part-time	27%
High School	10%	Employed, full-time	49%
Some College	20%	**Mobility Limitations**	**142**
College	32%	Yes	12%
Post-graduate	37%	No	88%

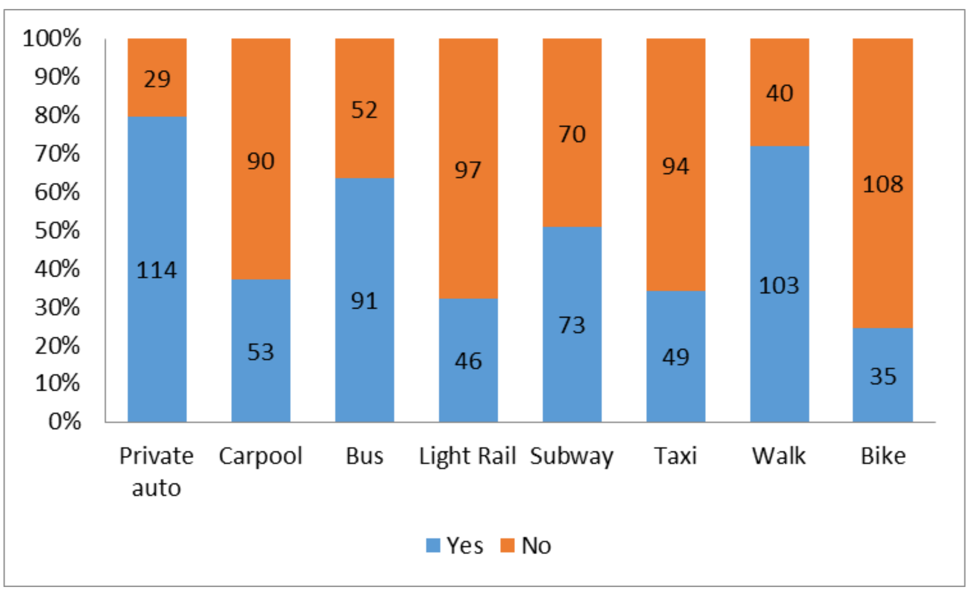


FIGURE 7 Modes taken in the past 6 months.

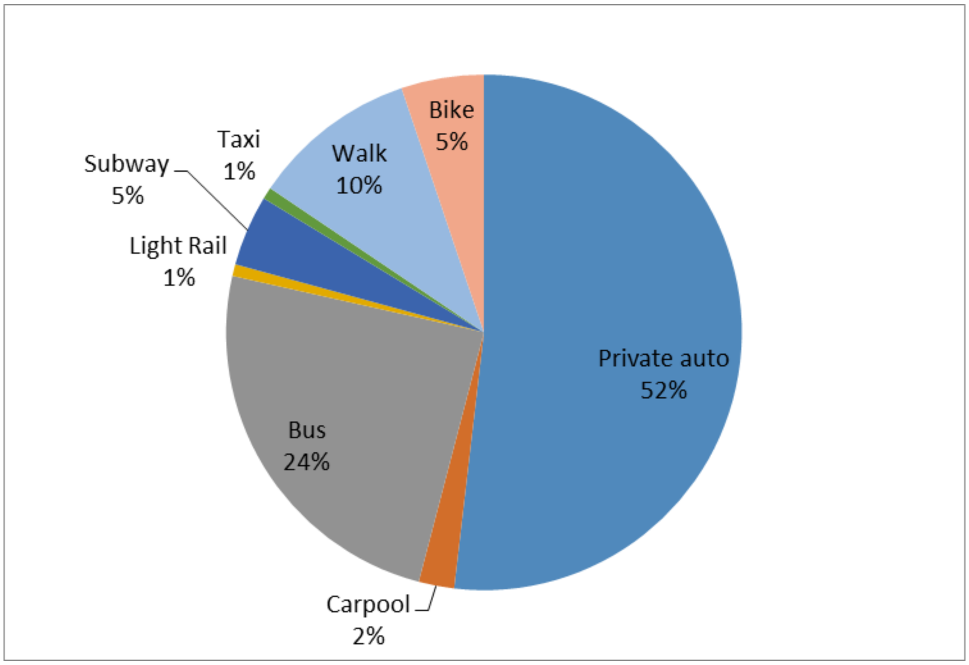


FIGURE 8 Primary mode over the past 6 months.

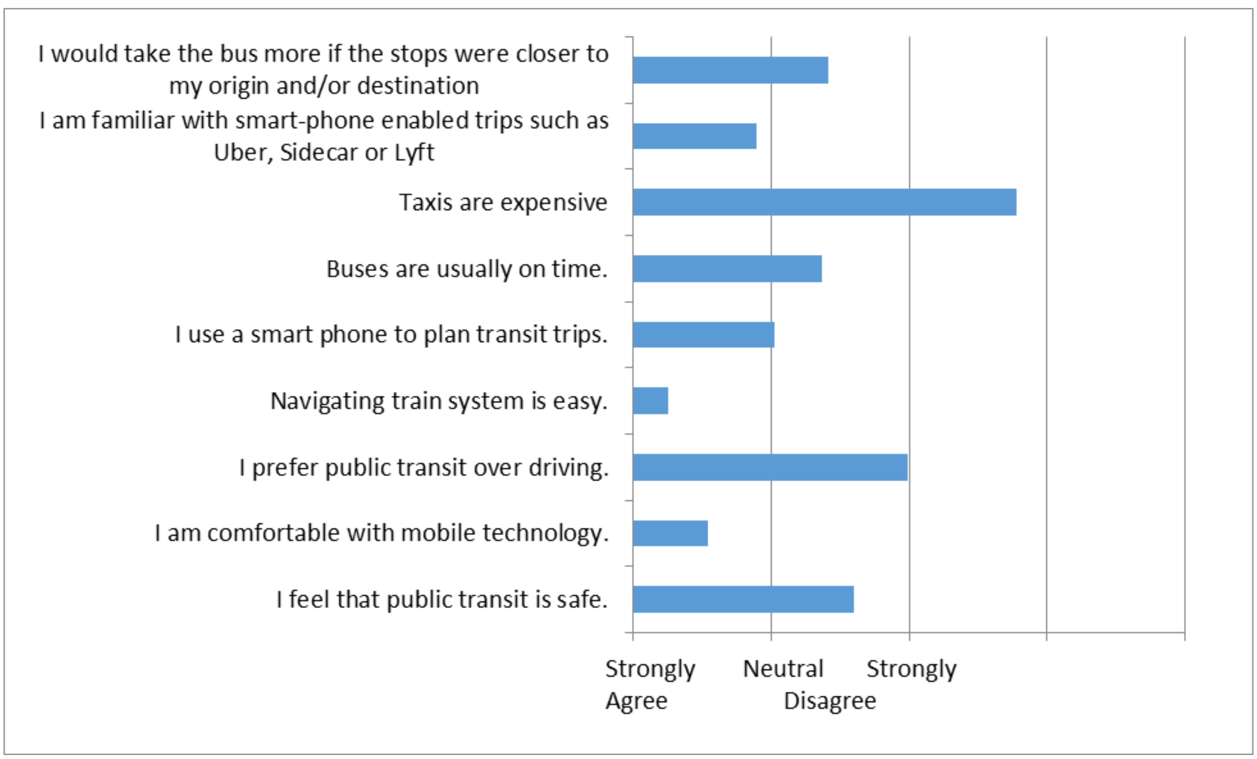


FIGURE 9 Mean Responses of Attitude Questions

The majority of respondents indicated private automobile as their primary mode of travel followed by bus. However, most the respondents had exposure to many modes of transport. The light rail category includes light rail, commuter rail and streetcars, and the taxi category includes taxis and transportation network companies (TNCs) such as Uber and Lyft and informal taxis known as Hacks in Baltimore. Though only 3% of respondents selected taxis and as the primary mode of transportation, over 30% of respondents had used each sometime in the past 6 months. This coupled with the fact that 63% of respondents have used the bus at some point in the 6 months prior to the survey suggest that demand responsive transportation would be considered as a transportation option.

FIGURE 4 shows the mean response of individual's attitudes toward transit systems and TNCs. On average, the respondents had favorable views towards transit. Most found it easy to navigate train systems, perceived bus as relatively on-time and felt moderately safe on transit systems. The respondents were tech savvy and were familiar with such services such as Uber and Lyft and utilize a smart phone when planning trips. People agreed that they would take the bus more if stops were located closer to home and were neutral to preferring driving over public transit. Lastly, most people felt that taxis were not expensive suggesting that the premium paid for door-to-door service is just.

MODELS

Each respondent was presented with 10 scenarios and asked to choose the most attractive mode between individual transportation (taxis, Uber), fixed-route service (traditional buses) and flexible-route service (paratransit, DRT). FIGURE 5 shows a summary of the responses received for these scenarios based on the overall selections. About 10% of the respondents always chose individual service, and there was always at least one scenario for each respondent for which individual service was deemed ideal. Not all options were selected: 31% of respondents never chose fixed-route service and 8% never chose flexible-route service.

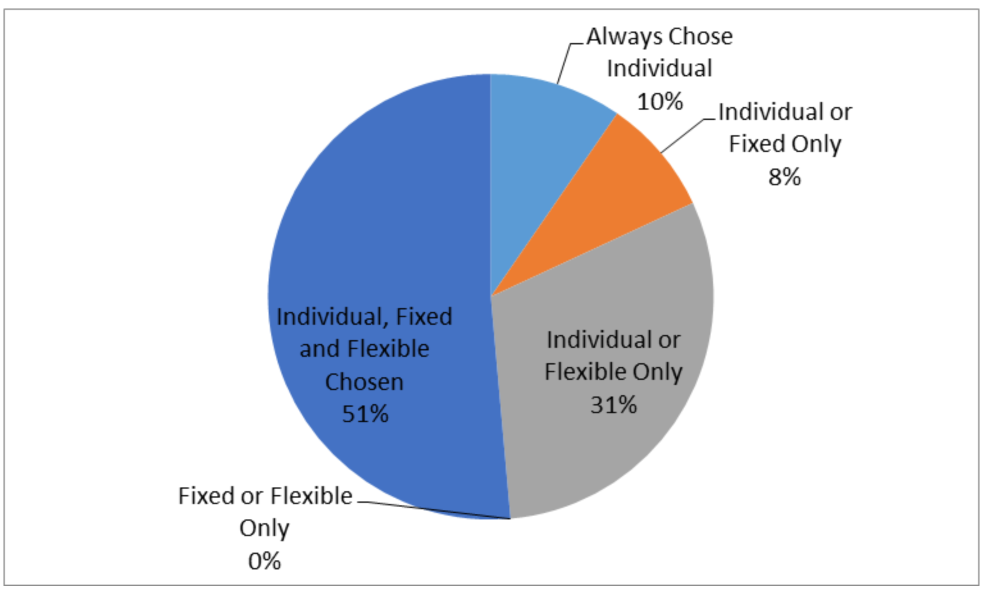
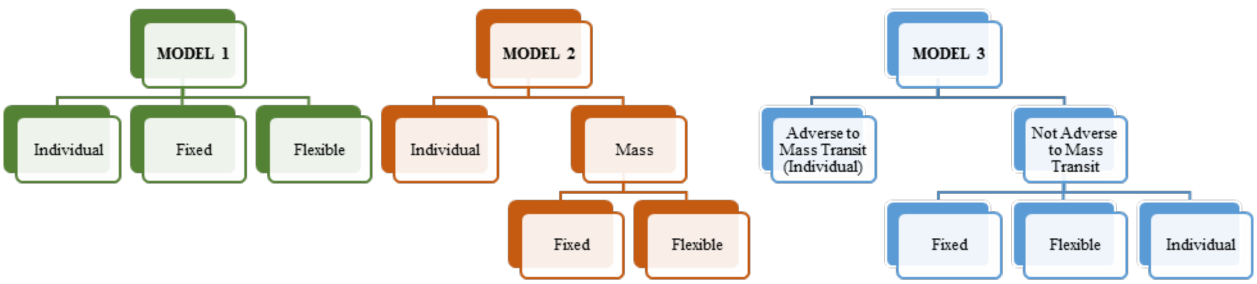


FIGURE 10 Summary of Choice Responses.

Three different modeling frameworks are proposed to describe how travelers select between the three travel options based on the attributes associated with each trip. Model 1 assumes that all respondents make one decision between three modes. Model 2 assumes that respondents make their choice in two steps: they first decide between individual and mass transportation; and, if mass transportation is chosen, they choose between fixed and flexible options. Lastly, Model 3 assumes that some portion of the population chooses to avoid mass transportation (i.e., fixed and flexible options) whereas those who do not then choose between fixed, flexible and individual transit. These three frameworks are visually depicted in FIGURE 6. Each model was developed considering the panel nature of the data; i.e., that the same respondent responds to 10 different scenarios presented. The models presented below were analyzed using BIOGEME 2.4 [13].



(a) (b) (c)

FIGURE 11 Choice Models: (a) Model 1 – Multi-nominal Logit, (b) Model 2 – Nested Logit, (c) Model 3 – Two Multi-nominal Logit Models.

Model 1

For this modeling framework, four individual multinominal logit models were developed using the form of (1). Each of these differs in the explanatory variables included in the model. Model 1a includes the alternative specific constants (α_j) and alternative explanatory variables ($\sum \beta_i x_i$) such as fare and travel times, Model 1b also includes the socioeconomic variables of each respondent ($\sum \gamma_j y_i$), and Models 1c and 1d includes respondent attitude variables towards transit ($\sum \delta_j z_i$). The model specification is as follows:

$$U_j = \alpha_j + \sum \beta_i x_i + \sum \gamma_j y_i + \sum \delta_j z_i + \epsilon_j \quad (1)$$

where

U_j = Utility of alternative j

α_j = Alternative specific constant of alternative j

β_i = Coefficient corresponding to explanatory variable x_i

γ_j = Coefficient corresponding to socioeconomic variable y_i

δ_j = Coefficient corresponding to attitudinal variable z_i

ϵ_j = Error term associated with alternative j

TABLE 23 Results of Choice Model 1 (1753 observations).

	Model 1a Adj R^2 =0.217 6 parameters		Model 1b Adj R^2 =0.227 12 parameters		Model 1c Adj. R^2 =0.232 15 parameters	
	Coeff, β_i	**p-value**	**Coeff, β_i**	**p-value**	**Coeff, β_i**	**p-value**
Alternative Specific Constants, α_j						
ASC_Fix	0.00	Fixed	0.00	Fixed	0.00	Fixed
ASC_Flex	-0.937	0.00*	-0.928	0.00*	-0.948	0.00*
ASC_Ind	-1.46	0.00*	-1.49	0.00*	-2.47	0.00*
Explanatory Variables, β_i						
Cost	-0.0630	0.00*	-0.0633	0.00*	-0.0641	0.00*
Avg. In-Veh Travel Time	-0.0205	0.01*	-0.0218	0.01*	-0.0220	0.01*
Mean Wait Time	-0.0109	0.00*	-0.0112	0.00*	-0.0113	0.00*
Walk Time	-0.0472	0.00*	-0.0481	0.00*	-0.0477	0.00*
Socioeconomic Variables, γ_j						
Age 25 and Under (Flex)			-0.296	0.03*	-0.296	0.03*
Age 25 and Under (Ind)			-0.369	0.03*	-0.489	0.01*
Car is Primary Mode (Ind)			0.277	0.06	0.239	0.11
Attended College (Flex)			-0.233	0.07	-0.217	0.09
Disabled (Flex)			0.832	0.00*	0.868	0.00*
Female (Flex)			0.350	0.00*	0.389	0.00*
Income Under 30k (Ind)					0.324	0.08
Attitudinal Variables, δ_j						
Feels Safe on Transit (Ind)					0.288	0.00*
Familiar with Uber (Ind)					0.300	0.06

The results of Model 1 are presented in TABLE 4; the asterisks denote significance at the 95% confidence interval. For all models panel effects were tested but proved insignificant. In Previous trials determined that having GPS on board, the range of in-vehicle travel times and the range of walking times were insignificant. The insignificance in the range of wait times suggests that people tend to focus more on average wait times in a stated preference environment despite research (e.g.[14]) showing the importance of reliability and uncertainty. Model 1b includes the same variables from Model 1a as well as socioeconomic variables. The fit of the model improved to an adjusted R-square of 0.227 in Model 1b as opposed to 0.217 in Model 1a. Respondents under the age of 25 were more likely choose fixed-route service than flexible-route service and choose flexible-route service over individual service. This may due to young person's aversion to high costs and both increased willingness and ability to walk further. Since private automobile was not a choice alternative, it is not surprising that noting private automobile as the primary mode of transport indicated an increased likeliness to choose the individual transit option. Those who attended college were more like to accept the new flexible route transit mode. Disable and female respondents are more likely to choose flexible route transportation.

Model 1c added attitudinal variables to Model 1b and the inclusion of Income Under $30,000, and the adjusted R-squared rose to 0.232. Being familiar with services such as Uber made one more likely to select the individual mode of transit.

Model 2

This framework had a nested structure in which a person first decides if they want to take the individual mode or a mass (shared) mode. Only if the individual chooses mass mode are they then assumed to decide between the fixed and flexible option. The nests are defined as:

$\lambda_{ind} = \{\text{individual}\}$

$\lambda_{mass} = \{\text{fixed, flexible}\}$

The Nested Logit model allows for the partial relaxation of independence of irrelevant alternatives (IIA). The Nested Logit model assumes that IIA holds within nests but not across nests.

A variety of models were tested using the nested model framework and the best model had the same independent variables as Model 1d. The values of the coefficients changed slightly and were generally consistent with intuition and the previous model results; see TABLE 5. However, this nested structure did not improve the overall fit of the model. Moreover, model parameters (λ), representing the branches of the upper level of the nest, were insignificant thus indicating that respondents weighed all three mode choices at once.

TABLE 24 Results of Choice Model 2 (1753 observations).

	Model 2 Adj. R^2 =0.232 15 parameters 1753 observations	
	Coeff, β_i	**p-value**
Alternative Specific Constants, α_j		
ASC_Fix	0.00	Fixed
ASC_Flex	-0.625	0.01*
ASC_Ind	-2.51	0.00*
Explanatory Variables, β_i		
Cost	-0.0560	0.00*
Avg. In-Veh Travel Time	-0.0163	0.02*
Mean Wait Time	-0.00799	0.01*
Walk Time	-0.0360	0.01*
Socioeconomic Variables, γ_j		
Age 25 and Under (Flex)	-0.206	0.06
Age 25 and Under (Ind)	-0.469	0.01*
Car is Primary Mode (Ind)	0.255	0.09
Attended College (Flex)	-0.154	0.12
Disabled (Flex)	0.612	0.00*
Female (Flex)	0.263	0.03*
Income Under 30k (Ind)	0.328	0.08
Attitudinal Variables, δ_j		
Feels Safe on Transit (Ind)	0.279	0.00*
Familiar with Uber (Ind)	0.302	0.05*
Model Parameters, λ_k		
Individual	1.00	Fixed
Mass	1.46	0.29

Model 3

This framework was considered to address the potential bias towards individual transportation modes in the stated preference survey. As shown in FIGURE 5, 10% of the respondents always chose the individual mode of transport in all 10 choice scenarios. Assuming this is a form of modal bias, this group of individuals may weaken the model. Thus, Model 3 separates the population into two segments: 1) those that always choose individual mode of transport; and, 2) those that consider and select between the three modal options available. The model is split into two models: respondent-level model and choice-level model.

Respondent-Level Model

The Respondent-Level Model examines the attributes that the respondents that are predictors of the choice to always choose the individual mode of transport. The model is alternative specific and only includes the socioeconomic and attitudinal variables of the respondents and does not include the cost and time values of the three modes since they would not matter in the decision-making process. A binary logit model is estimated as follows:

$$U_j = \alpha_j + \sum \gamma_j y_i + \sum \delta_j z_i + \epsilon_j \qquad (2)$$

where the alternatives j are now always choose individual mode (non-traders) and willing to trade off between modes (traders). For simplicity, the terms traders and non-traders are used.

The results of the Respondent-Level model are provided in TABLE 6. Several combinations of socioeconomic and attitudinal variables were considered. Generally, attitudinal variables were more significant that socioeconomic variables. Individuals whose primary mode of travel is the automobile were more likely to be non-traders. Conversely, traders had a more favorable view towards bus on-time performance and the safety of transit.

Choice Model for Traders

For the model that describes decision-making between the three travel options, two model forms were considered: Model 3-2a includes only the explanatory variables of the mode choices and Model 3-2b includes an additional six socioeconomic variables. There are fewer observations (1587) as we are only looking at non-traders. Thus, the fit of the model is slightly worse than Models 1 due to the limited sample size available. Overall, the model had coefficients that were in line with our expectations. Additional time and monetary costs, adversely affected ones mode choice. A respondent under the age of 25 was more likely to take fixed route service followed by flexible route service. This variable likely serves are a proxy a young person's individual's value of time. Females and those disabled are more likely to choose flexible route transportation.

Attitudinal variables were insignificant in predicting the mode choice of traders. However, one's view of on time performance and transit safety was an important factor in predicting if an individual is a trader or non-trader.

TABLE 25 Model 3 – Respondent Model

	Model 3-1 Adj R^2 =0.574 4 parameters 176 observations	
	Coeff, β_i	**p-value**
Alternative Specific Constants, α_j		
Traders (T)	0.00	Fixed
Non-Traders (NT)	0.863	0.32
Socioeconomic Variables, γ_j		
Car is Primary Mode (NT)	0.727	0.20
Attitudinal Variables, δ_j		
Bus are usually on time (T)	0.610	0.01*
Public Transit is Safe (T)	0.611	0.05*

TABLE 26 Model 3 – Traders Choice Model.

	Model 3-2a Adj R^2 =0.178 6 parameters 1587 observations		Model 3-2b Adj. R^2 =0.186 12 parameters 1587 observations	
	Coeff, β_i	**p-value**	**Coeff, β_i**	**p-value**
Alternative Specific Constants, α_j				
Fixed	0.00	Fixed	0.00	Fixed
Flexible	-0.773	0.00*	-0.782	0.00*
Individual	-1.28	0.00*	-1.34	0.00*
Explanatory Variables, β_i				
Cost	-0.0651	0.00*	-0.0653	0.00*
Avg. In-Veh Travel Time	-0.0210	0.01*	-0.0219	0.00*
Mean Wait Time	-0.0105	0.00*	-0.0108	0.00*
Walk Time	-0.0457	0.00*	-0.0470	0.00*
Socioeconomic Variables, γ_j				
Age 25 and Under (Flex)			-0.251	0.07
Age 25 and Under (Ind)			-0.349	0.04*
Car is Primary Mode (Ind)			0.329	0.02*
Attended College Flex (Flex)			-0.210	0.11
Disabled (Flex)			0.695	0.00*
Female (Flex)			0.335	0.01*

DISCUSSION

The results of the models presented above imply that the respondents analyzed the tradeoff of the three choice modes concurrently. Overall, the models show fairly consistent results. For example, the nested logit model (Model 2) was not statistically different from Model 1c. TABLE 8 compares Model 1 and Model 3. The first column shows the probability that the model predicts the correct mode choice of the respondents. The results indicate that including socioeconomic and attitudinal variables does not add much improvement to the models. This indicates that the parameters presented during the stated choice experiment (fare, in-vehicle travel time, waiting times) are most important predictors of transit mode choice. Model 3 overestimates the probability of people choosing individual mode of transportation at the expense of underestimate those who chose fixed route transportation resulting in a lower correct prediction rate. The poor performance indicates that always choosing the individual mode of transport does not imply that the respondent did not consider the other mode options. TABLE 8 also contains the estimated value of time for waiting and in-vehicle travel time obtained from these models. As can be seen, the models suggest that waiting time is valued about one-half as much as in-vehicle travel time. This seems inconsistent with previous research findings that suggest travelers value waiting time greater than in-vehicle travel. However, this result could be due to the stated preference nature of our survey. In the survey methodology, respondents are not actually subject to these waiting times and thus might not appropriately value them with respect to in-vehicle travel time. Nevertheless, since a comprehensive revealed preference mode choice model has not been created for the various transit options included here, these results reveal some additional insight into how transit users might select between traditional fixed, flexible and individual transit options.

TABLE 27 Comparison of Models 1 & 3

	% Correct Mode Choice	Value of Time	
		Waiting	In-Veh
Model 1a	65.3	$11.17	$19.84
Model 1c	65.8	$10.58	$20.59
Model 3a	62.3	$9.68	$19.35
Model 3b	62.7	$9.92	$20.12

Flexible route transit is unfamiliar to most individuals in the United States as it has largely been used for the transport of individuals with disabilities. This mode of transport allows for combines the economies of scale of fixed route transport with the flexibility of individual transportation modes. Predicting the adoption of an unfamiliar mode of transportation is challenging. This study presented a methodology for exploring the way respondents make mode choice decisions. It was found that socioeconomic and attitudinal variables did not greatly improve prediction models. If such flexible route systems are implemented, future work should compare the stated preference of an unfamiliar mode to revealed preferences post implementation.

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