Needs, Barriers and Analysis Methods for Integrated Urban Freight

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Needs, Barriers and Analysis Methods for Integrated Urban Freight Transportation

Final Report

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<td>In this joint project University of Maryland, West Virginia University, and Morgan State University worked together to solve critical problems associated with urban freight systems. A review of literature and case studies on freight villages and urban consolidation centers was conducted to identify factors critical for success of the strategies. Two specific problems were tackled in this research project. A novel mathematical model was developed to optimize transfers given the various sources of uncertainty in real world. The proposed probabilistic model is designed to minimize the expected cost and is generally applicable to cases including different distributions of random parameters. As an example application, the researchers have applied their model on the problem of dispatching trucks for a truck-rail intermodal system. The second problem tackled was accounting for retailer demand correlation in an joint location-inventory problem in an urban supply chain comprising of a plant, warehouse, and retailers. The resulting formulation is a convex mixed integer nonlinear program which can be efficiently solved using an outer approximation algorithm.</td>
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Acknowledgements

We would like to thank and acknowledge the Mid Atlantic University Transportation Center (MAUTC) and the United States Department of Transportation (USDOT) for funding this work. It was completed with the assistance of many individuals and organizations. The Principal Investigators wish to express thanks to those identified below, as well as all of the other individuals and organizations that supported the project.

Disclaimer

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1.0 Background

Ensuring a reliable goods delivery system is critical to the U.S. economy, and to our quality of life, as individuals try to meet their daily needs and insure access to goods and resources. While the growth in goods movements is experienced nationwide, urban areas are expected to feel the brunt of this increase. A major reason is that urban areas are themselves home to so many people and businesses that demand and produce the goods being moved. Furthermore, the increase in goods movement reflects the changing consumption patterns of consumers, such as the increased demand for a far greater array of products and home deliveries or Internet purchases.

There is a growing need to address the freight delivery issues in urban areas. Increasing truck traffic in urban areas will only add to the existing urban congestion problems, air pollution, and many other congestion-related externalities. To address such issues, urban delivery integration (UDI) strategies have been studied and adopted in many European and East Asian cities. UDI is defined for this study as a set of consolidated delivery schemes among businesses and other stakeholders to reduce logistics costs by increasing loading factors, mitigating congestion and helping to improve air quality. Under this approach, shipments from multiple carriers are consolidated into fewer full or near full trucks assigned to predefined routes, thus encouraging efficient use of truck capacity, as well as reduced road congestion. Moreover, the same amount of shipments is delivered with a smaller number of trucks and not with disjointed deliveries by several carriers. Despite empirical and conceptual findings, there is significant scope for improving the efficiency of such systems.
The problems addressed in this study are the difficulties, inefficiencies and negative impacts of freight deliveries in congested urban areas. Urban areas are highly dependent on efficient and reliable movements of goods, which include food and other supplies for urban residents, input materials and components for manufacturing firms and outbound shipments of products from urban producers. However, limited capacities of existing urban roads and limited availability of suitable places for loading and unloading freight vehicles (i.e., mostly trucks and vans) where needed, can greatly increase the cost and unreliability of deliveries. Furthermore, the activities of delivery vehicles can significantly add to the congestion, pollution, noise and other environmental problems in urban areas.

In this study, we first conduct a detailed literature review and case studies to identify critical factors affecting the success of freight villages and urban consolidation centers. We then focus on two problems associated with urban delivery systems. First is associated with identifying optimal locations and inventory management strategies of a multi-level urban supply chain. The second problem involves optimizing the transfers in multi-level freight systems which are again common in urban areas.

Problem Statement

The objective of this project is to:
1. Understand factors critical to the success of consolidation centers for urban freight delivery systems.
2. Develop an efficient model to optimize the location of warehouses and inventory control at warehouses in a multi-level supply chain with correlated retailer demand.
3. Develop a novel mathematical model for dispatching trucks that considers constraints and sources of stochasticity that arise in drayage operations and efficient heuristic based solution algorithms which solved the model.
2.0 Consolidation Centers Case Studies

2.1 Introduction

Freight movement is an indispensable part of the economy at various geographic levels. The reliability of freight delivery systems affects the flow of raw, intermediate, and finished goods among actors in a supply chain (e.g., suppliers, carriers, receivers, and consumers) that every part of the economy depends on to meet daily needs. Despite its integral role, freight movement—especially by truck—is often considered a public nuisance (Anderson, Allen and Browne 2005). It is difficult to balance business needs and the public perception of freight movement. A forecasted growth of population and the economy will increase both freight movement demand and conflicts between the two stakeholders. Even more alarming is that freight demand is growing a lot faster than population. The tonnage of freight shipment by all modes is forecast to increase by 45% between 2012 and 2040 (Federal Highway Administration 2014). During a similar time period (2014-2040), the U.S. population is expected to increase by 19% (Sandra and Ortman 2014, 6). Every one percent increase in population results in a more than two percent increase in freight demand. Moreover, truck dominance in goods movement will not be challenged in the foreseeable future. The Federal Highway Administration (FHWA) projected that in 2040, roughly 66% of the volume of goods would be transported by truck in the United States (Federal Highway Administration 2014). Considering over 80 percent of Americans live in urban areas (U.S. Census Bureau 2012), large cities and their metropolitan areas, especially where the built environment is dense, will feel the brunt of the increase. Continuing truck dominance in addition to freight demand growing faster than the population will only add to the existing urban air pollution, greenhouse gas emissions, noise pollution, and traffic safety issues (Browne, Allen, et al. 2012). At the same time, the reliability of freight...
delivery in cities will drop, which will increase unit logistics costs and decrease the competitiveness of the freight transportation sector and local economy.

To improve the reliability of urban freight delivery, various strategies have been studied and implemented in many countries, especially in Europe. Broadly, there are two strategies. The first involves promoting an efficient land use by clustering freight activities within designated areas near markets. Freight villages and urban consolidation centers (UCCs) are two widely known and implemented alternatives. A freight village is generally larger in land consumption than a typical UCC and is generally located in the suburbs of a city. On the other hand, a UCC is located next to a busy business/residential district within a city boundary or on the outskirts of a city. Smaller in scale than a freight village, UCCs often target a single large facility such as an airport, commercial district, or construction site. Both freight villages and UCCs require excellent access to various freight facilities. Freight consolidation is considered an ambitious, complex and large-scale alternative to meet two seemingly conflicting goals: ensuring economic competitiveness and delivery reliability and minimizing negative impacts from freight movement. It is believed that the agglomeration benefits of freight villages and UCCs decrease logistics costs, and consolidation requires fewer vehicles to complete delivery, thus decreasing freight trips. This improves environmental sustainability. The second strategy is proactive government involvement by providing appropriate rules and regulations to incentivize carriers conforming to government policies. Delivery time-window change, designated loading zones, parking restrictions, designated land use/building codes, and green logistics/vehicle decals, to name a few, are examples of government intervention. The two strategies are not mutually exclusive; thus, they should be considered in tandem. As discussed elsewhere in this report, two
of the factors in the successful implementation of freight villages and UCCs are government policy and political will.

For the purpose of this study as a whole, however, this report focuses on the first strategy. The following sections further discuss the general characteristics of freight villages and UCCs based on a scan of such facilities in practice, introduce interesting case studies, and contemplate the implications for application in the Baltimore Metropolitan Area.

2.2 Two Freight Consolidation Schemes

This section discusses the definitions and characteristics of freight villages and UCCs, with examples from European countries and Japan. Instead of focusing on individual facilities, we will mainly discuss interesting features found at the reviewed facilities and their implications.

2.2.1 Freight Villages

The term “freight village” is probably the most widely used around the world. In Europe and elsewhere, various terms such as transport center, logistics center, logistics park, Plateforme Logistique (Logistics Platform), Güterverkehrszentrum (Cargo Transport Center), and Interporto (Port Link) have been interchangeably used (Jarzemskis 2007). For this study, we borrowed the definition provided by EUROPLATFORMS:

*The hub of a specific area where all the activities relating to transport, logistics and goods distribution (...) are carried out (...) by various operators. (...) A Logistics Centre [i.e., freight village] must also be equipped with all the public [shared-use freight-related] facilities (...) If possible, it should also include public services for the staff as well as users’ equipment. In order to encourage intermodal transport for goods handling, a Logistics Centre [freight village]*
should preferably be served by a variety of transport methods (roads, rail, sea, inland waterways, air) (EUROPLATFORMS EEIG n.d., 3).

The primary goals of freight villages are (1) to maximize efficiency in goods movement and land use; and (2) at the same time to minimize the freight-related footprint in order to mitigate externalities such as tailpipe emissions, noise, congestion, and traffic safety issues (Bentzen, Hoffmann and Bentzen 2003). In concept, the two contradictory goals are achieved by efficient land use. That is, instead of letting freight businesses operate independently throughout an extended region along major freight corridors and nodes, all freight-related services and supporting services are located in a large designated area in the suburb of major market, in or near airports or seaports.

Various (often shared-use) freight facilities—i.e., transfer facilities for intermodal connections, warehouses, and distribution centers, and supporting services—are provided on site. This strategy decreases freight transportation flow, especially trucks, to and from independent facilities sprawled along transportation network, shifting the freight flow to the freight village. Goods shipped to the freight village are deconsolidated and consolidated shipments are transshipped to another mode depending on their destination. In doing so, the total number of freight transportation trips is reduced; thus, negative impacts of freight movement can be minimized and logistics costs can be reduced (SUTRANET 2007). What distinguishes freight villages from traditional freight facilities—freight terminals, industrial parks, inland ports, etc.—is that all necessary supporting services are clustered in a demarcated location and they can be shared among freight village tenants (Bentzen, Hoffmann and Bentzen 2003). In addition, administrative offices such as customs are located on site, saving required processing times for
imports and exports. To make this scheme feasible, in addition to the strong support of government entities, attracting a wide range of freight and logistics companies and providing easy access to highways, rails, air and/or water are key conditions (Liu and Savy 2013).

2.2.1.1 Freight Villages in Europe

It is believed that SOGARIS Logistics Centre in Rungis, France, opened in mid-1967, is the birthplace of the freight village (Kapros, Panou and Tsamboulas 2005). SOGARIS operates eight freight villages, called logistics platforms: seven in France and one in Luxemboug (SOGARIS n.d.). While SOGARIS is the only operating and management body, it was established as a public-private partnership; government agencies hold about 80% of shares.

Figure 1 shows three freight villages serving the Paris area. The Rungis Logistic Platform (number 1 in the figure) was opened first in 1967 as a truck terminal, just 7 km (4.3 miles) south of Paris. It has direct access to the A86, RN186, and RN7 motorways and is five minutes from Paris Orly Airport. It is right next to the Novatrans combined transport site and the Rungis International Market, the world’s largest fresh produce wholesale market. Some 80 companies—consisting of carriers, logistics service providers, forwarding agents, industrial companies, exporters/importers and distributors—are in business on 214,000 m² (52.9 acres) of warehouses, transit docks and offices. In 1992, the Roissy-SOGARIS Air Freight Logistics Platform (number 2 in the figure) started operating in freight area n.5 of Roissy Charles de Gaulle Airport, roughly 15 km (9.3 miles) northeast of Paris. Currently, 40 companies are operating, including freight and forwarding agents, express couriers, airlines, warehouses and airline spare parts companies and others. Supporting services such as on-site customs and inspection services are available. Lastly, the logistic platform of Créteil (number 3 in the figure) is the latest addition to the Paris
area. Opened in 2005, the facility is 6 km (3.7 miles) south of Paris and provides easy access to the Valenton combined rail-road transport hub. This system of three freight villages around a large metropolitan area is a good example applicable to large cities in the United States.

Figure 1  Freight Villages in Paris

Among freight villages in this case study, GVZ Grossbeeren is the largest in terms of the total land area, 759 acres, and equivalent to about 690 football fields or 4.7 Disneylands. The smallest among the cases is Roissy-SOGARIS, France, a 133-acre area where about 121 football fields can be built. Such huge land consumption prohibits a freight village location near the center of the city due to the scarcity of such a large area and high land rent. The establishment of “a neutral legal body” is considered one of the first steps in developing a freight village (EUROPLATFORMS EEIG n.d.). Public-private partnerships (PPPs) are the most common form of setting up these neutral parties so that the financial risk from such large-scale investments is
shared among participating sectors, both public and private (Jarzemskis 2007, EUROPLATFORMS n.d., Dablanc 2007). In one form of a PPP, the public sector ensures easy access to transportation infrastructure and public utilities, and provides a variety of incentives, while the private sector contributes the capital for constructing terminals, warehouses, distribution centers, and other related facilities. All five European LCFVs reviewed for this study (SOGARIS, France; Nordic Transportation Centre, Denmark; GVZ Bremen and GVZ Berlin-Brandenburg, Germany; and Interporto Bologna, Italy) were developed, managed, and/or operated by operating bodies based on public-private partnerships.

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<td>- Promote intermodal</td>
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**Table 1 Characteristics of Freight Villages: European Examples**

Source: C. de Cerreno et al. 2008
2.2.1.2 Freight Villages in the United States

There are many mega-scale freight facilities in the United States. For example, CenterPoint Intermodal Center (known as Joliet Arsenal), Elwood, IL, is situated on 3,000 acres, almost four times the size of GVZ Grossbeeren, the largest freight village reviewed for this study (JADA n.d.). The CenterPoint Intermodal Center specializes in rail-truck intermodal shipments. Adding more supporting services and commercial facilities makes this site similar to the freight villages defined in this study.

Probably the most well-known freight village in the U.S. is AllianceTexas. Built on 18,000 acres, AllianceTexas has created a “Freight City,” almost from scratch, on a scale never seen previously. AllianceTexas is the very example of the synergy of public-private collaboration. The strong support of the City of Dallas and the favorable business offer from Hillwood, the developer, convinced BNSF, a major railroad, to relocate to the area. “Given the scale of the development and the supporting infrastructure required it is obvious that local government, state agencies and federal agencies such as the FAA all facilitated and supported this development (Kirkland n.d.).” For example, a public-private partnership with the Texas Department of Transportation enabled the State Highway 170 construction. Figure 2 presents the spatial planning of AllianceTexas. Industrial, commercial, and residential land uses are closely located with moderate distances among each other. In particular, commercial activities (red pins on the figure) are functioning as a buffer between industrial land uses and residential areas. Currently, more than 400 companies are operating and over 40,000 are employed. This freight city includes commercial and residential areas as well as education and other public services (AllianceTexas n.d.).
2.2.2 Urban Distribution Centers

Another type of freight consolidation facility is the urban distribution center (UDC), also called urban consolidation center (UCC), city logistik, urban transshipment center, joint distribution center, etc. (Lin, Chen and Kawamura 2013). A UDC is defined (Panero, Shin and Lopez 2011):

A facility involving the trans-shipment of goods directed to urban areas, aiming to consolidate deliveries, and thus provide greater efficiency in the distribution process by increasing the truck load factor and decreasing the number of trucks used, which help mitigate urban congestion and air pollution.
While UDCs are considered to be a relatively recent concept, appearing in the 1990s in Europe, probably the earliest consolidation effort was implemented by the Port Authority of New York and New Jersey in the 1940s. But it stopped operating in the 1950s due to “union position and the lack of carrier participation” (Holguin-Veras, et al. 2015, 81). Another early consolidation that is still operating is a joint distribution center in Tenjin, Japan, opened in 1978.

UDCs are similar to freight villages in that freight is deconsolidated and consolidated in the shared-use facilities. However, locational characteristics and primary target areas distinguish UDCs from freight villages. The size of a typical UDC is much smaller than that of freight villages. For example, the London Construction Consolidation Center is just over 1 acre, and Interporto Padova, Italy, is about 7 acres (Interporto Padova) (Browne, Allen, et al. 2012, Interporto Padova SPA n.d.). This is because the primary goal of UDCs is to reduce the number of truck trips for last-mile delivery to a small target area. Thus, a more important requirement is the proximity to the target area where land costs are very high compared to the suburbs. Like freight villages, “communication, cooperation, and coordination among the various stakeholders” are critical conditions for successful implementation (Sinarimbo, Takahashi and Hyodo 2005). Consolidated shipments are delivered by a neutral carrier that specializes in the last-mile deliveries (Holguin-Veras, et al. 2015). By consolidating shipments and increasing load factor

Source: www.bestfact.net

Figure 3 Cargocycle
per truck, fewer trucks are needed to meet delivery demand within the UDC coverage area. To minimize environmental impacts, many UDCs use clean vehicles and/or non-motorized vehicles. For example, a fleet of hybrid and CNG vehicles are used in Interporto Padova, Italy (Cityporto n.d.). La Petite Reine, in France, uses cargocycles (Figure 3) to distribute parcels within the city center (La Petite Reine n.d.). The use of IT systems makes it possible to adjust delivery routes on a daily basis, depending on a total demand and destinations.

The review of over 40 UDCs in Europe revealed that strong government support and the involvement of all stakeholders from the beginning were key for success. For example, the German government led the national-level initiative of the UDC system with more than 70 cities in the 1990s (Browne, et al. 2005). Not all were actually implemented, but Germany established one of the most comprehensive and coordinated networks of UDCs and freight villages. Switzerland and the Netherlands soon adopted similar initiatives modeled after the German plan (BESTUFS 2007, 117, McKinnon 1998).

A rare case in terms of the championing of the UDC initiative is the joint delivery system in Tenjin, Fukuoka, started in 1978 with the support of the Ministry of Transport (Taniguchi and Qureshii 2014, Nemoto 1997). The uniqueness lies in the initiation of joint delivery by 29 local freight carriers that delivered goods to Tenjin District, a business/commercial center of Fukuoka (Taniguchi 2002). The limited loading/unloading zones in the densely built area led to ubiquitous illegal parking that only added to existing congestion. The collaboration started to address increasing logistics costs (Browne, Sweet, et al. 2005). The operating body changed in 1994 to the Tenjin District Joint Distribution Company Ltd., established by 36 companies and responsible for goods delivery and collection services to and from the Tenjin 1st Street to 5th Street area (Taniguchi, Tenjin Joint Distribution System, Fukuoka, Japan 2002). Carriers unload
shipments at the Hakozaki Joint Distribution Center in the suburb of Fukuoka City (Figure 4). After sorting and consolidating parcels, carriers affiliated with the Center make deliveries to receivers. The member carriers also collect parcels on the way back, minimizing empty return trips.

Source: Taniguchi 2002

Figure 4 Map of Tenjin District and Joint Distribution Center

La Petite Reine in Paris provides another unique feature that can be applicable in busy downtown areas in large U.S. cities. La Petite Reine is operating on a 6,460 ft² space; it provides room for short-term storage, sorting, loading, and warehousing (Conway, et al. 2012). About a quarter of a million parcels are delivered annually in Paris in addition to collection services. Conway et al. (2012) pointed out three success factors of this private-sector initiative. First, in collaboration with the public sector, cargocycles are able to access restricted areas such as pedestrian zones and narrow urban streets. Second, cargocycles can park anywhere. Finally, due to congestion in Paris, cargocycles have a travel time advantage in this short-distance delivery
market. La Petite Reine offers a good example of how a private venture can be self-sustainable. In addition to its own business model, branding it as a socially and environmentally responsible company attracted customers (Panero, Shin and Lopez 2011).

2.2.3 Benefits of Freight Villages and Urban Distribution Centers

Freight villages have lots of benefits. First, freight villages and UDCs have the potential to address congestion (Bentzen et al., 2003; Kapros et al., 2005). However, the congestion mitigation effect of freight villages is somewhat anecdotal and needs more real-world evidence. In the case of UDCs, a number of references provided real-world evidence. For example, in Friedberg, Germany, a UDC with 12 participating transportation companies reduced the number of trucks by 51 percent, reduced delivery trips by 33 percent, and saved delivery time by 48 percent (Visser et al., 1999). In Italy, 24 months operation of Cityporto UDC reduced total truck VMT by 348,838 miles, and the average daily reduction was 756 miles per day (Cityporto n.d.). In a case study in Sweden, the use of consolidated delivery to a convention center in 2008 decreased weekly truck trips from 400 trips to 20 trips (Nilsson 2009 cited in Holguin-Veras et al. 2015, 81).

Second, fewer trucks and reduced truck VMT means less air pollution. During the 24-month period in Cityporto Padova, Italy, CO\textsuperscript{2} emissions were reduced by 220 tons, and a significant reduction of other pollutants such as NOx, Sox, VOC, and PM10 was observed (Cityporto n.d.).

Third, by locating multiple freight and industrial activities in the same area, instead of having them scattered throughout a region, efficient land use becomes possible (Wagener 2008). Many freight facilities owned by a single company are usually small in size and sprawled along major highways. A landscape of warehouses, distribution centers, and other freight facilities
along a 7-mile stretch between Exit 12 and Exit 15E of the New Jersey Turnpike for several miles is a typical example. Attracting these facilities to a single freight village or a network of freight villages and UDCs prevents excessive land consumption.

Fourth, freight villages and UDCs promote economic development in the surrounding communities as well as to the tenants. As discussed earlier, AllianceTexas alone hires over 40,000 employees. A case study of Greek freight villages found the return on investment was 24.6 percent (Kapros, Panou and Tsamboulas 2005).

2.2.4 Factors for successful implementation

Reviewing cases for this report revealed factors for successful implementation. All cases showed that a strong support of the public sector is a key in addition to the collaboration with all affected stakeholders from the beginning of the discussion. Stakeholder involvement is especially important since freight facilities and UDCs need a certain number of business participants to be sustainable. In this sense, the public sector should champion this initiative and ensure the private sector understands urban sustainability and its benefits to their delivery operation. For example, in the cases of SOGARIS, France, and Interporto Bologna, Italy, public agencies are the largest shareholders of the operating body. Germany championed the national-level planning and policy to rationalize freight distribution networks. More importantly, the role of the public sector as a mediator and facilitator is particularly critical. Improving delivery reliability, promoting economic development, and mitigating negative externalities are not easy to achieve at the same time. In this sense, the public sector should give something to every stakeholder. This, more often than not, cannot be realized without government rules and regulations and incentives.
By the same token, it should be noted that not all freight village or UDC initiatives were successful or are still in business. Freight consolidation strategies show a mixed success since participating in a freight village or UDC is not necessarily efficient and cost effective to certain freight industry (Kawamura and Lu 2007). In addition, in many large urban areas, especially given the extensive urban sprawl in the United States, securing suitable space at reasonable land rent would be difficult (Browne, Sweet, et al. 2005). As a result, public subsidy is a critical requirement at least at the setting-up stage.

2.3 Selected Case Study: Two Tiered System

2.3.1 Berlin-Brandenburg Capital Region’s Network of Freight Villages and UDCs

Throughout this literature review of freight village and UDC case studies, the study team tried to identify an applicable form of freight consolidation. We found that a German model of a network of freight villages and UDCs would be appropriate in the United States and the Baltimore metropolitan area, in particular.

Germany has one of the most extensive and planned network of freight villages and urban logistics facilities. According to a 2008 study (Wagener 2008), at least 31 freight villages are in operation in which 1,300 companies are operating (Figure 5). In particular, the Berlin-Brandenburg Capital Region has promoted the region as logistics cities (Wagener 2008). As of 2015, six freight villages (GVZs) and 18 UDCs (Figure 6) constitute the regional system that has easy access to two airports, highways and rail, and where about 205,000 are employed (ZAB Brandenburg Economic Development Board 2015, 2-3). In addition, the collaboration with more than 13 universities and research institutions strengthens the innovative force in the network. The primary goal of the cluster is to promote the economic competitiveness of the region through green logistics and intermodality. According to Bentzen et al. (2003), this is “a model of how
Freight Villages can be integrated and can play a key role within a complex environment for solving urban freight distribution.” This system is expected to solve “last-mile” issues by reducing the number of truck trips for urban distribution. When the first UDC was built, about 6,000 truck trips were saved per year (Bentzen, Hoffmann and Bentzen 2003).
Source: (ZAB Brandenburg Economic Development Board 2015)
2.3.2 Implications for Applicability in the Baltimore Region

The City of Baltimore serves several of the largest markets in the nation such as Washington, D.C., Philadelphia, and Delaware. Given the development patterns of the city for the past several decades, more development of office buildings, townhouses and condominiums will be concentrated along the harbor, which is served by rail and Interstates 70, 83, 95, 395, 695, and 895. The area is also home to freight rails and trucks traveling to and from the city and the Port of Baltimore and destined for nearby markets, as well as through traffic along I-95, one of the busiest freight corridors in the United States. With that said, the region is facing the daunting task of striking a balance between demands for reliable freight movement by trucks and meeting the needs of other surface transportation modes and residential and business (re)development. Moreover, the opening of the expanded Panama Canal is expected to give locational advantages to the mid-Atlantic ports, including the Port of Baltimore, over the geographically constrained Port of New York and New Jersey, which has more limited transportation access. In Baltimore, the cost-prohibitive alternative of adding more rail and increasing the height of the Howard Street Tunnel does not seem to be a reliable alternative, continuously disconnecting freight rail services coming from the East and the West. This means that the city needs to rationalize the flow of long- and short-distance shipments by truck. The model of Berlin-Brandenburg is a good benchmark for the city. As shown in Figure 7, Baltimore and Washington, D.C., are closely located and, economically and geographically, the area as a
whole is one large metropolitan area. The major freight corridors connecting the two cities are I-95, I-695, and I-495, known for day-long traffic congestion. A freight village in the middle of the city and another freight village exclusively serving the Port of Baltimore would divert significant truck volumes from the three interstate highways, especially truck traffic through the two large cities. In addition, small-scale UDCs could be located on the outskirts of the two cities that are connected to the freight village and a neutral carrier – i.e., a carrier not related to shippers and receivers - account for the last leg of the supply chain. Borrowing from La Petite Reine, France, and Interporto Padova, non-motorized vehicles and clean vehicles could be used for last-mile delivery. A more radical idea such as reusing vacant buildings as drop and pick-up depots could be considered when enough density is ensured.

Source: Google Earth Pro
Figure 7 Baltimore and Washington D.C. Metropolitan Areas

From the policy perspective and for planning purposes, the Morgan State team found three types of facility operations for integrated freight delivery: public-private partnerships, public-owned facility, and private-owned facility. Regardless of the operation types, the most important tasks are to identify potential sectors of industry and provide some types of incentives to attract private businesses. In addition, the findings from the literature review and a series of meetings suggest that a large-scale integrated freight delivery center may not be feasible. Instead, an integration targeting a small geographic area, building complex (e.g., mall), or a single business sector (e.g., restaurant) would be more feasible. Also, stakeholder meeting participants pointed out that the benefits of participation should be clearly presented to the private sector.

2.4 Strategies to overcome barriers for implementation

Ironically, the success factors mentioned in an earlier section are daunting barriers for implanting a consolidation strategy. The first barrier is to identify the private businesses that are willing to participate in the consolidation initiative. Parcel carriers such as UPS, FedEx, USPS, and DHL have established their own networks and many companies use their services for the last-mile delivery. And big box groceries like Wal-Mart, Target, Safeway and others have their own optimized system. Having them participate in the consolidation initiative would be extremely difficult, if not impossible, without strong mandatory policies that may not be feasible in the United States in the foreseeable future. Due to these limitations, focusing on small shippers, carriers, and receivers that would benefit from consolidation is the best strategy. This strategy would make sense given the extremely diverse types of goods and businesses moved in cities (Dablanc 2005). Starting at a smaller scale and targeting a smaller area/sector (e.g., restaurants or a large mall) is a feasible strategy. According to two separate surveys conducted in
New York City and California, 16-18 percent of carriers answered they are likely to participate in a consolidation initiative (Holguin-Veras, Silas and Polimeni 2008, Regan and Colob, Trucking industry demand for urban shared-use freight terminals n.d.). Such a percentage is not insignificant at all.

Another barrier is government support. Many once-successful initiatives closed down or plans were not implemented due to financial difficulties after the government stopped providing subsidies (Dablanc 2005). Moreover, freight transportation companies had less incentive to use UDCs because subcontracting to a small carrier was often cheaper than using UDCs (Dablanc 2005). Just arguing for sustainability does not make the case to the private sector. Cost sharing and strong enforcement/disincentives to non-participating shippers, carriers, and receivers are necessary elements.

Third, acceptance and understanding of the public-sector planners are also important. A smaller scale survey of public-sector freight stakeholders in Maryland shows that just over 14 percent (3 of 21 responses) of responses moderately favored freight consolidation. On the other hand, 38 percent favored route restriction and 33 percent favored delivery time-window change, including off-hour delivery. These alternatives are not mutually exclusive strategies; when they are implemented together, the synergy will be significant.

Fourth, an interesting but potentially promising alternative was suggested by the Dutch researchers (van Rooiien and Quak 2010 cited in Holguin-Veras et al, 2015). They suggested that convincing receivers to participate in a consolidation program is more effective. Since receivers are the ones who determine delivery time windows and often delivery methods, this suggestion is worth considering.
2.5 Summary

This chapter discussed the concepts of freight villages and UDCs. Also discussed are characteristics of these consolidation facilities. The review of European cases and one Japanese case provides insights on the application of the strategies in the Baltimore region. Especially, given the geographic and economic circumstances of the region, a network of freight villages and UDCs implemented in the Berlin-Brandenburg Capital Region in Germany was suggested as a benchmark. To be successful, collaboration among public and private stakeholders from the beginning is essential. More importantly, the role of government as a facilitator and coordinator should be emphasized. The review of past surveys and a survey of Maryland public stakeholders supported three types of strategies: route restriction, time-window change, and consolidation. When these strategies are implemented at the same time, the effectiveness of freight consolidation strategy will increase significantly. However, given the complexity of the supply chain and the requirement of huge initial capital, focusing on a small geographic scale or a specific sector was suggested to get the initiative started.
3.0 Three Level Location-Inventory Problem with Correlated Demand

3.1 Introduction

Supply chain networks serve as the basis of operation for many industries. In today's competitive market, with risky uncertain operational environment it is highly important to design such networks in a cost-effective, efficient, and responsive manner (Friesz et al. 2011). According to Harrison (2004) an appropriate network design strategy can potentially decrease the cost of a supply chain by 60%. In the classical setting, design of such systems is primarily based on standalone treatment of strategic problems involving the number and location of required facilities (warehouses and plants) for a network with minimum cost without effectively addressing operational side problems (such as inventory control and level of service). Typically, in such a framework the operational decisions are made after the locations are determined. Nevertheless, as discussed by many researchers, this method does not yield the most effective network structure and can increase the redundancies among different echelons of a supply chain (see, e.g., Miranda and Garrido, 2004; Snyder et al., 2007). Therefore, joint inventory facility location problems were introduced, in which strategic and operational problems are solved in an integrated framework capturing connections among retailers, distributors, suppliers, and other supply chain entities. Daskin et al. (2002), Shen et al. (2003), and Miranda and Garrido (2004) were among the first who mathematically modeled the joint facility location and inventory control problems. Their models involved the specification of location of warehouses and the customer's allocation while also optimizing inventory decisions. They also considered safety stock cost as a risk pooling strategy to cope with demand uncertainty at the retailer level while assuming a single plant with constant lead time.
Accounting for the safety stock cost in a multi-echelon supply chain network tends to be more complex, especially with the changes occurring in supply chain management due to new practices such as coordination (Arshinder & Deshmukh, 2008). An example of such changes is the increase in demand correlation over the time and space as a result of information sharing between the retailers of a supply chain networks (Lee et al., 2000). Several studies have focused on the value of modeling demand correlation in supply chain management (see, e.g., Ganesha et al., 2014; Helper et al., 2010; Raghunathan, 2003; So and Zheng, 2003; and Güllü, 1997). Park et al. (2010) studied the three level location-inventory problem where the safety stock cost accounts for plant location dependent lead time; however, their model did not consider the impact of demand correlation across the retailers. This study extends the work of Park et al. (2010) by incorporating demand correlation in a three level supply chain network design model where lead time is dependent on the plant location. Accounting for demand correlation can better represent the risk pooling strategy within a joint inventory location problem (Snyder 2006).

In particular, this study's contributions are as follows: first, we present a three level location-inventory problem with correlated demand which simultaneously minimizes the total cost for three types of decisions: (i) location of warehouses and plants, (ii) assignment of warehouses to the plants and the assignment of retailers to the warehouses, and finally, (iii) optimal inventory level and safety stock cost at the warehouses. Second, we demonstrate that the initially-proposed binary nonlinear integer program (BNIP) formulation can be transformed into a mixed integer conic quadratic program (MICQP). This transformation is performed to exploit the advances made by solvers such as CPLEX in solving second order conic integer programs. An outer approximation-based algorithm is used to solve the model.
3.2 Literature review

This section provides a review of the integrated facility location and inventory control problem, considering risk-pooling benefits. The classical literature on joint location-inventory control problems can be categorized into works which considered the capacity restriction on the facilities and the papers without any capacity assumption. We first describe these two types, followed by a discussion of the works on integrated facility location and inventory control problem in multi-echelon networks and the literature on the reliability based design of such networks.

Most of the early works in this area studied the uncapacitated case. Daskin et al. (2002) proposed a nonlinear integer programming model in which the objective is to minimize transportation, location, and nonlinear safety stock costs. A number of heuristic approaches based on Lagrangian relaxation were employed to solve the problem. Shen et al. (2003) reformulated the Daskin et al. (2002) model as a set covering model and used a column generation based procedure to efficiently solve the model for two special cases: first, when the variance of demand is proportional to the mean, and second, when the variance of demand is zero. Shu et al. (2005) developed a column generation approach that relaxes the two special cases of the above work to a general instance utilizing certain special structure of the pricing problem. Teo and Shu (2004) extended the above work by determining the optimal inventory policies for DCs and retailers using an infinite horizon two-echelon inventory cost function. The problem was formulated as a set-partitioning integer programming model and solved using column generation. Shu and Sun (2006) and Snyder et al. (2007) studied the scenario-based stochastic variant of the models developed by Daskin et al. (2002) and Shen et al. (2003). Snyder et al. (2007) considered the two special cases on demands identified by Shen et al. (2003) and
employed a Lagrangian relaxation algorithm to solve the problem. However, Shu and Sun (2006) reformulated the problem as a set covering model, and provided a column generation solution approach which is not restricted to the two special cases of demand scheme.

Capacity restriction on facilities is a practical assumption which has been incorporated into the joint inventory framework by many researchers. Miranda and Garrido (2004) combined inventory control decisions with a capacitated facility location problem (CFLP). Their model locates DCs, assigns retailers to them, and defines the best inventory policy at each DC, considering DC capacity limitations. The resulting nonlinear mixed integer problem was solved using Lagrangian relaxation and sub-gradient methods. Miranda and Garrido (2006) extended the previous work by considering two types of capacity constraints on DCs. The first one limits the maximum order quantity, while the second sets a maximum inventory level for each DC. Romeijin et al. (2007) studied the capacitated version of the model proposed by Teo and Shu (2004). They formulated the problem as a set covering model and used a column generation approach. The relation between lead time and safety stock was undertaken in a study by Sourirajan et al. (2007) where the proposed model appreciates the tradeoff between lead times and inventory risk-pooling benefits. While Sourirajan et al. (2007) employed a Lagrangian heuristic approach to solve the problem; Sourirajan et al. (2009) used a genetic algorithm which can also be extended to incorporate arbitrary demand variance at retailers. The capacitated version of the works by Daskin et al. (2002) and Shen et al. (2003) was studied by Ozsen et al. (2008) where a Lagrangian relaxation approach was used as a solution methodology. Atamtürk et al. (2012) studied several reformulated variants of a joint inventory location problem as a mixed-integer conic quadratic program (MICQP). The proposed modeling framework can be solved using commercial optimization packages which lead to promising computational solution times.
While Atamtürk et al. (2012) considered a two-echelon network; this study proposes a three-echelon supply chain model.

Many studies have examined the joint location inventory problems in a multi-echelon framework with various operational and practical assumptions. Vidyarthi et al. (2007) proposed a multi-product case of joint inventory location model that locates plants and DCs, determines the amount of shipments from plants to DCs and assigns retailers to DCs. However, they only considered safety stock costs as the risk management strategy. Park et al. (2010) discussed the influence of plant locations and their assigned DCs on the amount of safety stock maintained at DCs and provided a three-level supply chain network model in which DC-to-supplier dependent lead times are considered. A two-phase heuristic solution algorithm based on the Lagrangian relaxation method was derived. Tancrez et al. (2012) proposed a three-layer model that accounts for some practical features such as direct shipments and undertaking inventory in each layer. The proposed model was solved using a simple heuristic that can be applied to large supply networks design. Keskin and Üster (2012) proposed a three-level production-distribution system design considering inventories at DCs and retailers, and capacity constraints at plants. An efficient heuristic approach combining a local search technique and simulated annealing algorithm was developed to solve the problem.

A four-level supply chain network design problem was proposed by Shahabi et al. (2013) in which they investigated the effect of hub facilities within a multi-echelon facility location problem with inventory and risk pooling strategies.

Accounting for reliability and risk mitigation strategies in supply chain networks is another growing body of literature which has been tackled by number of researchers. Friesz et al. (2011) studied the influence of disruption on supply chain flows in a competitive environment.
Masih-Tehrani et al. (2011) considered dependent uncertainty in supplier behavior in a two echelon supply chain network in which the effect of various factors such as disruption level, dependency of disruptions, and customer demand were investigated on the total system performance and policy settings. A simulation based approach for modeling disruption in transportation links for multi-echelon supply chain network was proposed by Schmitt (2011) where transportation links are subject to failure. The results showed that the customer level of service can be improved by a combination of inventory and back up strategy. The reliable design of supply chain networks in the face of disruption in facilities is another avenue of research for mitigating risk in supply chain networks which has been discussed in many papers. Wang and Ouyang (2013) studied the competitive facility location and supply chain network design with disruption whereas Chen et al. (2011), Peng et al. (2011) and Li and Ouyang (2010) considered the location design problem with facilities subject to disruption. In addition, Lu et al. (2011) exploited the product substitution strategy under the multiple sourcing as a risk mitigation technique in order to deal with the failures on the supplier’s side.

In this study, we consider a three-echelon supply chain network with warehouse-to-supplier-dependent lead time, considering correlated retailer demands in order to better model the safety stock cost as the risk pooling strategy. To the best of our knowledge, Atamtürk et al. (2012) is the only other work to incorporate demand correlation for design of multi-echelon supply chain network design. While Atamtürk et al. (2012) considered a two-echelon network; this study proposes a three-echelon supply chain model. In addition, we propose an outer approximation algorithm as the solution methodology which is highly efficient for this type of program.
3.3 Problem definition

This section presents the mathematical programming formulation for the three-level location-inventory problem considering demand correlation among retailers. In particular the formulation involves three different decisions: (i) a multi-level facility location problem to determine the number and location of plants and warehouses, (ii) an allocation problem to determine the best assignment of retailers to located warehouses and located warehouses to located plants, and (iii) inventory control decisions at each located warehouse. The aim of the model is to simultaneously minimize the facility location, transportation and the inventory costs incurred by the network in the presence of correlation between the retailers demand.

Specifically, the model adopts the following assumptions:

- There is a fixed setup cost for opening plants and warehouses.
- The transportation cost per unit shipment, both between plants and warehouses and between warehouses and retailers, is proportional to the Euclidean distance.
- A single sourcing strategy is considered, in which each retailer is supplied only from a single warehouse, and each warehouse is sourced only from a single plant.
- Inventory control is considered only at warehouses, and follows a continuous review inventory policy \((r, Q)\), i.e., once the inventory level at a warehouse falls below a reorder point \(r\), the fixed quantity \(Q\) is ordered from the appropriate plant.
- A safety stock is held at each warehouse to cope with demand variations of its assigned retailers.
- Demands at retailers follow a multivariate normal distribution with a known mean vector and covariance matrix.
• There is a plant-to-warehouse lead time which depends on the location of the plant and the warehouse.

• Stockout costs are not considered in this work, and the service level is given.

• Plants are not subject to any capacity limitations, while a finite handling capacity is considered at each warehouse.

Furthermore, the model indices, parameters, and variables used throughout this chapter are presented below:

Indices

\( i, l \)
Index for retailers \((1, ..., R)\)

\( j \)
Index for warehouses \((1, ..., W)\)

\( k \)
Index for plants \((1, ..., P)\)

Parameters

\( g_k \)
Annual fixed setup cost for plant \( k \) ($)

\( f_{jk} \)
Annual fixed cost of locating warehouse \( j \) assigned to plant \( k \) ($)

\( t_{jk} \)
Per-unit transportation cost between plant \( k \) and warehouse \( j \) ($)

\( c_{lj} \)
Per-unit transportation cost between warehouse \( j \) and retailer \( i \) ($)

\( A_j \)
Per order fixed inventory ordering cost at warehouse \( j \) ($)

\( h_j \)
Per unit per year inventory holding cost at warehouse \( j \) ($)

\( Cap_j \)
Daily capacity for warehouse \( j \)

\( r_j \)
Optimal reorder level at warehouse \( j \)

\( \mu_i \)
Mean daily demand at retailer \( i \)

\( \sigma_i^2 \)
Variance of daily demand at retailer \( i \)

33
\( \rho_{il} \) Correlation coefficient between daily demands at retailer \( i \) and at retailer \( l \)

\( l_{jk} \) Order lead time in days from plant \( k \) to warehouse \( j \)

\( \eta \) Number of working days per year

\( z_\alpha \) \( \alpha \)-percentile of the standard normal distribution

**Variables**

\( D_j \) Mean daily demand at warehouse \( j \)

\( U_j \) Variance of daily demand at warehouse \( j \)

\( Q_j^* \) Optimal order quantity at warehouse \( j \)

\( L_j \) Order lead time at warehouse \( j \) (day)

\( SS_j \) Safety stock level at warehouse \( j \)

\( TC_j^{INV^*} \) Optimal Inventory Cost at warehouse \( j \)

**Decision Variables for Optimization Formulation**

\( v_k \in \{0,1\} \) Takes value 1 iff a plant is located at \( k \)

\( y_{jk} \in \{0,1\} \) Takes value 1 iff warehouse \( j \) is assigned to plant \( k \)

\( x_{ij} \in \{0,1\} \) Takes value 1 iff retailer \( i \) is assigned to warehouse \( j \)

The demands at retailer \( i \) and retailer \( l \) are correlated with a known correlation coefficient \( \rho_{il} \). The daily demand at warehouse \( j \) is assumed to follow a multivariate normal distribution with mean \( D_j = \sum_i \mu_i x_{ij} \) and variance \( U_j = \sum_i \sum_l \rho_{il} \sigma_i \sigma_l x_{ij} x_{lj} \). In addition, the order lead time at warehouse \( j \) can be obtained as \( L_j = \sum_k l_{jk} y_{jk} \). Then, the demand at warehouse \( j \) during the lead time is normally distributed with mean \( D_j L_j \) and variance \( U_j L_j \).
3.3.1 Formulation

In this study, we adopted the optimal order quantity and reorder level of \((r, Q)\) based on the classical economic order quantity approximation method which has been extensively used in location problems inventory (e.g., Daskin et al. 2002; Shen et al. 2003; Miranda & Garrido, 2004). Based on this method, the optimal ordering quantity \(Q_j^*\), the optimal reorder level \(r_j\) and the safety stock level \(SS_j\) at each warehouse \(j\) considering retailers correlated demand and plant dependent lead time are obtained as:

\[
Q_j^* = \sqrt{2A_j \eta D_j / h_j} = \sqrt{2A_j \eta \sum \mu_i x_{ij} / h_j} \quad (1)
\]

\[
r_j = D_j L_j + z_\alpha \sqrt{U_j L_j} \quad (2)
\]

\[
SS_j = z_\alpha \sqrt{U_j L_j} = z_\alpha \sqrt{\sum \sum \rho_{il} \sigma_i \sigma_l x_{ij} x_{lj} \sum l_{jk} y_{jk}}
\]

\[
= z_\alpha \sqrt{\sum \sum \sum \rho_{il} \sigma_i \sigma_l l_{jk} x_{ij} x_{lj} y_{jk}} \quad (3)
\]

where \(\alpha\) is the highest allowable occurrence probability for stockout during the lead time, which is commonly called a “service level” and \(z_\alpha\) is the \(\alpha\)-percentile of the standard normal distribution. The optimal inventory cost function at each warehouse can be obtained as equation (4):

\[
TC_j^{INV*} = \sqrt{2A_j h_j \eta D_j} + z_\alpha h_j \sqrt{U_j L_j} = \sqrt{2A_j h_j \eta \sum \mu_i x_{ij} + z_\alpha h_j \sqrt{\sum \sum \sum \rho_{il} \sigma_i \sigma_l l_{jk} x_{ij} x_{lj} y_{jk}}} \quad (4)
\]
The mathematical programming formulation for the multi-echelon facility location and inventory control model is then:

P1:

\[
\begin{align*}
\text{Min} & \quad \sum_k g_k v_k + \sum_j \sum_k f_{jk} y_{jk} + \sum_i \sum_j \sum_k \eta \mu_{t_{jk}} x_{ij} y_{jk} + \sum_i \sum_j \eta \mu_i c_{ij} x_{ij} \\
& + \sum_j \sqrt{2A_j h_j} \eta \sum_i \mu_i x_{ij} + \sum_j z_{\alpha} h_j \sum_i \sum_k \rho_i \sigma_i \sigma_{l_{jk}} x_{ij} x_{ij} y_{jk} \\
& \quad \text{subject to:} \\
& \quad \sum_j x_{ij} = 1, \quad \forall i \tag{6} \\
& \quad x_{ij} \leq \sum_k y_{jk}, \quad \forall i, j \tag{7} \\
& \quad \sum_i \mu_i x_{ij} \leq \text{Cap}_j, \quad \forall j \tag{8} \\
& \quad \sum_k y_{jk} \leq 1, \quad \forall j \tag{9} \\
& \quad y_{jk} \leq v_k, \quad \forall j, k \tag{10} \\
& \quad x_{ij}, y_{jk}, v_k \in \{0,1\} \quad \forall i, j, k \tag{11}
\end{align*}
\]

In the above formulation, the objective function (5) minimizes the total annual cost which is comprised of the total cost of locating facilities, transportation, and inventory cost. Equations (7) and (10) ensure that each retailer and each opened warehouse will be assigned to an opened
warehouse, and an opened plant, respectively. Equations (6) and (9) express the single sourcing strategy assumption for each retailer and each warehouse. Equation (8) enforces the capacity constraint at each warehouse.

The proposed model (P1) for three level facility location and inventory control problem is an extension of the model proposed by Park et al. (2010) in which retailers’ demand is assumed to be correlated. Considering correlation among retailers demand is one step forward in incorporating risk pooling effect in distribution network design which is mathematically expressed in the safety stock cost considered in the last term of the objective function. Considering demand correlation significantly increases the difficulty of solving the formulation, as the number of nonlinear terms would significantly increase, thus posing a challenge for existing solution methods in the literature.

In particular, model (P1) is difficult to solve because it is a binary nonlinear integer program (BNIP) with complicated nonlinear terms in the objective function. However, (P1) can be transformed and reformulated as a mixed integer conic quadratic program (MICQP) which is a more tractable formulation. More details on the reformulation have been presented in the next subsection.

3.3.2 Mixed Integer Conic Quadratic Programming Formulation

In the reformulation scheme presented in this section we first replace the quadratic term $x_{ij} y_{jk}$ by introducing a new nonnegative variable $M_{ijk}$ and adding three new constraints to the formulation. This linearization scheme is popular for solving quadratic assignment problems. However for the problem studied here, the linearization results in a significantly high number of constraints. Therefore, we try to reduce the size of added constraints by adopting a reduction strategy in order to gain efficiency. The three new constraints are:
\[ M_{ijk} \leq x_{ij} \quad \forall i, j, k \quad (12) \]
\[ M_{ijk} \leq y_{jk} \quad \forall i, j, k \quad (13) \]
\[ M_{ijk} \geq x_{ij} + y_{jk} - 1 \quad \forall i, j, k \quad (14) \]
\[ M_{ijk} \geq 0 \quad \forall i, j, k \quad (15) \]

Thus the above equations effectively constrain \( M_{ijk} \) to be 1 if \( x_{ij} = y_{jk} = 1 \), and 0 otherwise. (Note that \( M_{ijk} \) does not require an explicit integrality constraint.) Preposition 1 allows us to replace constraints (12)–(15) by an equivalent set of constraints which is smaller in size.

**Proposition 1.** Given any feasible solution \((x, y, v)\) to \((P1)\), the set of feasible \( M \) defined by constraints (12–15) is identical to the set of \( M \) defined by the constraints:

\[ \sum_k M_{ijk} = x_{ij} \quad \forall i, j \quad (16) \]
\[ M_{ijk} \leq y_{jk} \quad \forall i, j, k \quad (17) \]
\[ M_{ijk} \geq 0 \quad \forall i, j, k \quad (18) \]

**Proof.** Assume that \((x, y, v)\) is a feasible solution to problem \((P1)\), and that \( M \) satisfies (12)–(15). Consider any retailer \( i \). By feasibility to \((P1)\) there is exactly one \( j^* \) and exactly one \( k^* \) such that \( x_{ij^*} = 1 \) and \( y_{j^*k^*} = 1 \). By (14), \( M_{ij^*k^*} = 1 \), and by (12) and (13) \( M_{ijk} = 0 \) for any other \( j \) and \( k \). Thus (16) is satisfied. Conditions (17) and (18) are simply conditions (13) and (15), and thus \( M \) satisfies (16)–(18) as well.
Conversely, assume that $M$ satisfies (16)–(18), and again consider any retailer $i$ assigned to warehouse $j^*$ and plant $k^*$. Let $j$ be arbitrary. If $j \neq j^*$, then $x_{ij} = 0$ and (16) implies $M_{ijk} = 0$ for all $k$, satisfying (12) and (14). If $j = j^*$, then $x_{ij} = 1$, so $M_{ijk} \geq 0$ for at least one $k$. However, $y_{j^*k}$ is only nonzero for $k = k^*$, so (17) enforces $M_{ij^*k} = 0$ unless $k = k^*$, so (16) implies $M_{ij^*k^*} = 1$; in all cases (12) and (14) are satisfied. Conditions (13) and (15) are simply (17) and (18), and thus $M$ satisfies (12)–(15). QED

In particular, since conditions (12)–(15) implicitly enforce an integrality constraint on the $M$ values, Lemma 1 implies that conditions (16)–(18) do as well. This is a major advantage, since it helps to reduce the size of the search space in the branch-and-bound tree.

In addition, since $y_{jk}$ is a binary integer variable we can rewrite $x_{ij}x_{ij}y_{jk}^2$ in the objective function as $x_{ij}x_{ij}y_{jk}^2$ since the term $y_{jk}$ can be substituted by $y_{jk}^2$ and thus $x_{ij}x_{ij}y_{jk}^2$ can be expressed in terms of the new variable as $M_{ijk}M_{ijk}$ by adding constraints (16)–(18) to the formulation. Furthermore, by introducing new positive variables $t_{1j}$ and $t_{2j}$ and taking into account that $x_{ij} = x_{ij}^2$ for the binary variable $x_{ij}$ the MICQP counterpart for model P1 is presented as below:

\[ \text{P2:} \]

\[ \begin{align*}
\text{Min} & \quad \sum_k g_k v_k + \sum_j \sum_k f_{jk} y_{jk} + \sum_i \sum_j \eta \mu_i t_{jk} M_{ijk} + \sum_i \sum_j \eta \mu_i c_{ij} x_{ij} \\
& \quad + \sum_j t_{1j} + \sum_j z_{\alpha} h_j t_{2j} \\
\text{subject to:} & \quad \left(19\right) \\
& \quad \sqrt{2 A_j h_j \eta} \sum_i \mu_i x_{ij}^2 \leq t_{1j} \\
& \quad \forall j \quad \left(20\right) 
\end{align*} \]
\[
\sum_{i} \sum_{l} \sum_{k} \rho_{il} \sigma_{l} l_{jk} M_{ij} M_{lk} \leq t_{2j} \quad \forall j
\]

\[
\sum_{k} M_{ijk} = x_{ij} \quad \forall i, j
\]

\[
M_{ijk} \leq y_{jk} \quad \forall i, j, k
\]

\[
t_{1j}, t_{2j} \geq 0 \quad \forall j
\]

\[
M_{ijk} \geq 0 \quad \forall i, j, k
\]

Eqs. (6) - (11)

Model P2 has a linear objective function and conic quadratic constraints (20) and (21). Reformulating model (P1) as MICQP (P2) would enable us to directly employ optimization solvers like CPLEX. However, the efficiency of these solvers would decrease rapidly with increase in network size, because the correlation matrix grows quadratically with the number of retailers, making computation of constraints (21) expensive. Therefore an outer approximation algorithm has been developed in which the nonlinear terms in the original formulation are approximated by linear constraints through an iterative process.

In next section the concept and details of the OA approximation as the solution strategy for the proposed formulation is presented.

3.4 Solution Methodology

Outer approximation, a technique originally proposed by Duran & Grossman (1986), is a method for solving MINLPs. OA belongs to the family of cutting plane algorithms which operates by iteratively solving a series of nonlinear programs (NLP) as a subproblem, and mixed integer linear programs (MILP) as a master problem. Bonami et al. (2008) proved that OA will
deliver a global optimally solution for convex mixed integer nonlinear programs. A MINLP is a convex MINLP if the continuous relaxation of the initial formulation is convex. Given a feasible integer assignment for the master problem, the OA subproblem is to solve the NLP in which all the integer variables are fixed. The convexity of the nonlinear terms allows for linear approximations which then provide a master problem with MILP structure. At each iteration of the algorithm the master problem is enhanced with finer approximations of the nonlinear terms. The solutions of the subproblem due to feasibility to the primary problem provide an upper bound for the algorithm, whereas solutions to the master problem yield lower bounds. The algorithm iteratively cycles between subproblem and the master problem until the difference between these bounds is sufficiently small. The readers may refer to Duran & Grossman (1986) and Fletcher & Leyffer (1994) for a comprehensive description of the OA framework.

Before applying OA, we need to prove that the continuous relaxations of the functions defining constraints (20) and (21) are convex in order to assure optimality and applicability of the OA approach. Lemma 2 establishes this property.

**Proposition 2.** The functions $F(x_{ij}, t_{1j}) = \sqrt{2A_j h_j \eta \sum_{i} \mu_i x_{ij}^2 - t_{1j}}$ and $G(M_{ijk}, t_{2j}) = \sqrt{\sum_l \sum_k \sum_i \rho_{il} \sigma_i \sigma_{lk} M_{ijk} M_{ljk} - t_{2j}}$ defined on the domain $[0, 1] \times \mathbb{R}$, are convex.

**Proof:** By linearity, it suffices to show that the radical term is convex. For all $j$ we have

$$\sqrt{2A_j h_j \eta \sum_{i} \mu_i x_{ij}^2} = (2A_j h_j \eta)^{0.5} \sqrt{\sum_{i} (\mu_i x_{ij})^2} = (2A_j h_j \eta)^{0.5} \|X\|$$
where \(\|\cdot\|\) is the Euclidean norm and \(X\) is a vector in the form of \(X = (\sqrt{\mu_1 x_{1j}}, \ldots, \sqrt{\mu_i x_{Rj}})\), therefore it suffices to show that \(\|X\|\) is convex. But for any vectors \(X_1\) and \(X_2\), the triangle inequality implies
\[
\|(\lambda X_1 + (1 - \lambda)X_2)\| = \|\lambda X_1 + (1 - \lambda)X_2\|
\leq \|\lambda X_1\| + \|(1 - \lambda)X_2\|
= \lambda \|X_1\| + (1 - \lambda) \|X_2\|
\]
which establishing the convexity of \(F\). Similarly for \(G\), it is enough to show convexity of each radical term. Let \(Q\) denote the covariance matrix. Since the covariance matrix is positive semi-definite, the Cholesky decomposition exists and there is a matrix \(B\) such that \(Q = BB^T\). Thus we can write:
\[
\sqrt{\sum_i \sum_l \sum_k \rho_{il} \sigma_i \sigma_l l_{jk} M_{ijk} M_{ijk}} = \sqrt{\sum_i \sum_l \sum_k l_{jk} M_{ijk} B_{il} B_{il} M_{ijl}} = \|M_B\|
\]
where the vector \(M_k\) is defined as \(M_k = (\sqrt{l_{jk} M_{ijk}}, \ldots, \sqrt{l_{jk} M_{ijk}})\). As before the triangle inequality establishes the convexity of this function, completing the proof. QED

3.4.1 Outer Approximation Subproblem

Given values for the integer decision variables, the OA subproblem finds the optimal value for the continuous variables, providing a feasible point in order to approximate the nonlinear constraints (20) and (21). In OA algorithm, the subproblem is typically the algorithmic bottleneck because it requires solving an NLP at each iteration. However, in the case of model P2 the optimal value for the continuous variables \(t_{1j}\) and \(t_{2j}\) can be obtained in closed form, through equations (27) and (28). Furthermore, the values of the continuous variable \(M_{ijk}\) can be directly calculated from the binary variables \(x_{ij}\) and \(y_{jk}\) which are fixed in the subproblem.
Therefore, the NLP subproblem can be trivially solved. More specifically, given a feasible integer solution $x_{ij}^h, y_{jk}^h, \nu_k^h$ at every iteration $h$, the optimal values for continuous variables $\bar{M}_{ijk}^h, \hat{t}_1^h$ and $\hat{t}_2^h$ (and hence the OA upper bound) are:

$$\bar{M}_{ijk}^h = x_{ij}^h y_{jk}^h \quad \forall i, j, k$$

$$(26)$$

$$\hat{t}_{1j}^h = \sqrt{2A_j h \eta \sum \mu_i x_{ij}^h} \quad \forall j$$

$$(27)$$

$$\hat{t}_{2j}^h = \sqrt{\sum \sum \sum \rho_{il} \sigma_i l_{jk} \bar{M}_{ijk}^h \bar{M}_{ijk}^h} \quad \forall j$$

$$(28)$$

$$Z^h = \sum g_k \nu_k^h + \sum f_{jk} y_{jk}^h + \sum \mu_i t c_{ijk} \bar{M}_{ijk}^h + \sum \eta \mu_i c_{ij} x_{ij}^h$$

$$+ \sum \hat{t}_{1j}^h + \sum z_{\alpha h} j \hat{t}_{2j}^h \quad \forall j$$

$$(29)$$

Generally, OA requires an initial integer feasible solution to be started. In the case of the proposed problem which is the integration of facility location and inventory control decision problem, solving a multi-level capacitated facility location problem provides an initial integer feasible solution to begin the algorithm.

3.4.2 Outer Approximation Master Problem

We can build the OA master problem provided that the optimal values for variables $x_{ij}^h, y_{jk}^h, \nu_k^h, \hat{t}_1^h, \hat{t}_2^h$ and $\bar{M}_{ijk}^h$ at every iteration $h$ is available. Proposition 3 shows how the linear approximations of the constraints (20) and (21) are calculated.
Proposition 3.

If \( \hat{x}_{ij}^h, \hat{y}_{jk}^h, \hat{v}_k^h, \hat{t}_{1j}^h, \hat{t}_{2j}^h \) and \( \bar{M}_{ijk}^h \) are an optimal solution for the nonlinear subproblem of the OA algorithm at iteration \( h \), the following two inequalities are valid linear outer approximation cuts for the constraints (20), and (21) respectively:

\[
2A_j h \eta \sum_i \mu_i \hat{x}_{ij}^h (x_{ij} - \hat{x}_{ij}^h) - \hat{t}_{1j}^h (t_{1j} - \hat{t}_{1j}^h) \leq 0 \quad \forall j, h \quad (30)
\]

\[
\sum_l \sum_i \rho_i \sigma_l \sum_k (M_{ijk} - \bar{M}_{ijk}^h) + \sum_l \sum_i \rho_i \sigma_l \sum_k (M_{ijk} - \bar{M}_{ijk}^h) - 2\hat{t}_{2j}^h (t_{2j} - \hat{t}_{2j}^h) \leq 0 \quad \forall j, h \quad (31)
\]

Proof:

Consider \( F(x_{ij}, t_{1j}) \) as a function where its continuous relaxation is convex. Given a feasible assignment point of \( (\hat{x}_{ij}^h, \hat{t}_{1j}^h) \) at iteration \( h \) for the problem P2, by convexity of \( F(x_{ij}, t_{1j}) \) we have:

\[
F(x_{ij}^h, t_{1j}^h) + \nabla F(x_{ij}^h, t_{1j}^h)^T [x_{ij} - x_{ij}^h]_{t_1 - t_{1j}^h} \leq F(x_{ij}, t_{1j}) \leq 0 \quad (32)
\]

Therefore, setting \( F(x_{ij}, t_{1j}) = \sqrt{2A_j h \eta \sum_i \mu_i x_{ij}^2 - t_{1j}} \), the linear approximation provides

\[
\sqrt{2A_j h \eta \sum_i \mu_i \hat{x}_{ij}^h + \frac{2A_j h \eta \sum_i \mu_i \hat{x}_{ij}^h}{2\sqrt{2A_j h \eta \sum_i \mu_i \hat{x}_{ij}^h}}} (x_{ij} - \hat{x}_{ij}^h) - \hat{t}_{1j}^h - (t_{1j} - \hat{t}_{1j}^h) \leq 0 \quad \forall j, h \quad (33)
\]

Furthermore, given the optimal solution of \( \hat{t}_{1j}^h = \sqrt{2A_j h \eta \sum_i \mu_i \hat{x}_{ij}^h} \) at every iteration \( h \) from equation (27) we can further simplify equation (33) as the following:
\[
\frac{A_j h_j \eta \sum_i \mu_i \hat{x}_{ij}^h}{\sqrt{2A_j h_j \eta \sum_i \mu_i \hat{x}_{ij}^h}}(x_{ij} - \hat{x}_{ij}^h) - (t_{1j} - \hat{t}_{1j}^h) \leq 0 \quad \forall j, h \tag{34}
\]

Also, since \(\hat{t}_{1j} = \sqrt{2A_j h_j \eta \sum_i \mu_i \hat{x}_{ij}^h} \) and \(\hat{t}_{1j}^h \neq 0\), pre-multiplying equation (34) by \(\hat{t}_{1j}^h\) would result in expression (35) and the proof is complete. QED

\[
2A_j h_j \eta \sum_i \mu_i \hat{x}_{ij}^h (x_{ij} - \hat{x}_{ij}^h) - \hat{t}_{1j}^h(t_{1j} - \hat{t}_{1j}^h) \leq 0 \quad \forall j, h \tag{35}
\]

In addition, expression (38) can also be achieved through the same procedure. Finally, given the above linear approximations, the master problem for OA can be given as below:

OA MP:

\[
\text{Min } \sum_k g_k v_k + \sum_j \sum_k f_{jk} y_{jk} + \sum_i \sum_j \sum_k \eta \mu_i t_{jk} M_{ijk} + \sum_i \sum_j \sum_k \eta \mu_i c_{ij} x_{ij} + \sum_j t_{1j} + \sum_j z_{a} h_j t_{2j}
\]

subject to:

\[
2A_j h_j \eta \sum_i \mu_i \hat{x}_{ij}^h (x_{ij} - \hat{x}_{ij}^h) - \hat{t}_{1j}^h(t_{1j} - \hat{t}_{1j}^h) \leq 0 \quad \forall j, h \tag{37}
\]

\[
\sum_i \sum_j \sum_k \rho_{il} \sigma_i l_{jk} M_{ijk}^h (M_{ijk} - \hat{M}_{ijk}^h)
\]

\[
+ \sum_i \sum_j \sum_k \rho_{il} \sigma_i l_{jk} \hat{M}_{ijk}^h (M_{ijk} - \hat{M}_{ijk}^h) - 2\hat{t}_{2j}^h(t_{2j} - \hat{t}_{2j}^h) \leq 0 \quad \forall j, h \tag{38}
\]

\[
\sum_{i,j,k \in S^h} M_{ijk} - \sum_{i,j,k \in S^h} \sum_{i,j,k \in S^h} M_{ijk} \leq |S^h| - 1 \quad \forall h \tag{39}
\]

\[
f^h < Z^h - \epsilon \quad \forall h \tag{40}
\]

Eqs. (6) - (11)

Eqs. (22) - (25)
The OA algorithm involves repeatedly solving the MILP master problem where at every iteration $h$ constraints (37) and (38) are added to the problem to linearly approximate the convex nonlinear feasible region of model P2. The addition of constraints (37) and (38) at every iteration to the MP of the OA algorithm will provide a better approximation of the convex feasible region which guarantees the convergence of the algorithm for convex MINLP (Bonami et al. 2008). Equation (39) is an integer cut which guarantees that the integer solutions at every iteration $h$ is different from the previous iterations. In this equation, $S^h = \{(i,j,k) : \tilde{M}_{ijk}^h = 1\}$ and $S'^h = \{(i,j,k) : \tilde{M}_{ijk}^h = 0\}$ at every iteration $h$. Moreover, $|S^h|$ denotes the cardinality of $S^h$ at iteration $h$. In addition, Fletcher & Leyffer (1994) included equation (40) in the structure of the master problem to ensure that the solution of the master problem is less than the upper bound at every iteration $h$. In their proposed framework, the algorithm can be terminated whenever the solution to master problem is infeasible, at which the solution is $\varepsilon$-optimal. In this scheme, the master problem need not be solved to optimality as long as a new feasible integer solution for the master problem is obtained. Following the procedure proposed by Fletcher & Leyffer (1994), the algorithm is illustrated in Figure 8.


**Outer Approximation (OA)**

**Input:** OA optimality gap, maximum number of iterations $H$

**Initialization:** Set $UB = +\infty$ and $LB = -\infty$, $h = 1$

Solve a multi-level capacitated facility location problem for initial value of $\hat{y}_{ijk}^1$, $\hat{v}_k^1$, $\hat{x}_{ij}^1$ and $\hat{M}_{ijk}^1$

While (Master problem is feasible and $h \leq H$)

  1: Calculate the continuous variables $\hat{M}_{ijk}^h$, $\hat{r}_1^h$ and $\hat{r}_2^h$ according to equations (26), (27) and (28)

  2: Calculate the subproblem objective function $Z^h$ according to equation (29)

  3: If ($Z^h < \text{UP}$), update the upper bound and update the current best points as $\bar{y}_{jk} = \hat{y}_{jk}^h$, $\bar{x}_{ij} = \hat{x}_{ij}^h$, $\bar{v}_k = \hat{v}_k^h$, $\bar{M}_{ijk} = \hat{M}_{ijk}^h$, $\bar{r}_1 = \hat{r}_1^h$ and $\bar{r}_2 = \hat{r}_2^h$

  4: Add equations (37)–(40) for iteration $h$ and solve the MP based on equation (36)–(40), (6)–(11) and (22)–(25).

  5: Update the LB, increment $h$.

**Report the solutions:** Report $\bar{y}_{jk}$, $\bar{x}_{ij}$, $\bar{v}_k$, $\bar{M}_{ijk}$, $\bar{r}_1$ and $\bar{r}_2$.

---

3.5 Numerical Experiments

In this section three sets of computational experiments are presented to study the performance of the proposed OA algorithm on networks with varying size. In the first set of experiments we compare the performance of the OA based solution strategy with the performance of BARON and CPLEX solvers in solving mixed integer nonlinear programming formulation $P1$ and mixed integer conic quadratic programming formulation $P2$ respectively. In addition, the second sets of experiments are designed to investigate the importance of incorporating the demand correlation within the structure of location-inventory problems. Finally, the third sets of experiments study the computational performance and sensitivity of the OA algorithm to different parameters such as warehouse capacity, the holding cost and correlation coefficient. The tests were carried out on a 3.4 GHz Dell Optiplex 990 Pentium i7-2600
computer with the 64-bit version of the Windows 7 operating system with 8 GB RAM. The OA algorithm was coded in GAMS, and CPLEX 12 was used to solve the linear master problem of the algorithm.

3.5.1 Experiment Setup

The test networks in this study have been generated randomly based on the strategy presented in Park et al. (2010). The potential locations of the plants, warehouses, and location of retailers are randomly generated according to a uniform distribution over the square of \((0,10]\). Also, the transportation costs between each pair of plant and warehouse and warehouse and retailer are assumed to be based on kilometer and proportional to the Euclidean distances between the generated locations, with the cost ratio information based on US industries presented in Nozick & Turnquist (2001). More specifically, per unit distance per unit demand transportation costs are set at $0.5 and $1.00 for inbound and outbound network respectively.

Defining \(\mu_0\) as the base mean retailer demand, \(\sigma_0\) as the base demand standard deviation at each retailer, \(C_0\) as the base warehouse capacity, \(f_0\) as the base fixed cost of locating warehouse and \(h_0\) as the base holding cost, the required equations for generating the network and model parameters have been presented in Table 2. In this table, \(\bar{c}\) is the average value of the per unit transportation cost between warehouses and retailers, \(\theta\) is the percentage of the cost variation, \(\bar{\mu}\) is the average value of the mean retailer demands, and finally \(\bar{f}\) is the average value of \(f_{jk}\)’s for all \(j\) and \(k\). It is worth noting that the data are generated in a manner that the inventory control decisions may affect the network design decisions (Park et al., 2010). Furthermore, the test problems, which consist of different combinations of plants, warehouses, and retailers, are presented in Table 3 providing size of the test problems considered in this study.
Table 2 Data Generation

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean daily demand</td>
<td>$\mu_i = \mu_0 \cdot U[1,5]$</td>
</tr>
<tr>
<td>Daily demand standard deviation</td>
<td>$\sigma_i = \sqrt{\sigma_0^2 \cdot U[1,5]}$</td>
</tr>
<tr>
<td>Demand correlation coefficients</td>
<td>$\rho_{ii} = 0.5, \ i \neq l$ &amp; $\rho_{ll} = 1, \ i = l$</td>
</tr>
<tr>
<td>Warehouse capacities</td>
<td>$Cap_j = C_0 \cdot \bar{\mu} \cdot (1 + U[-\theta, \theta])$</td>
</tr>
<tr>
<td>Annual fixed cost of locating warehouse $j$ assigned to plant $k$ ($$$)</td>
<td>$f_{jk} = f_0 \cdot \bar{c} \cdot Cap_j \cdot (1 + U[-\theta, \theta])$</td>
</tr>
<tr>
<td>Annual fixed cost of locating plant ($$$)</td>
<td>$g_k = 2 \cdot \bar{f} \cdot (1 + U[-\theta, \theta])$</td>
</tr>
<tr>
<td>Fixed ordering setup costs per order ($$$)</td>
<td>$A_j = A_0 \cdot \bar{c} \cdot \bar{\mu} \cdot (1 + U[-\theta, \theta])$</td>
</tr>
<tr>
<td>Per unit per year holding costs ($$$)</td>
<td>$h_j = h_0 \cdot (1 + U[-\theta, \theta])$</td>
</tr>
<tr>
<td>Order lead times (day)</td>
<td>$l_{jk} = 3 \cdot U[1,5]$</td>
</tr>
</tbody>
</table>

Table 3 Test Networks

<table>
<thead>
<tr>
<th>Problem Instance</th>
<th>Total Number of potential Plants (P)</th>
<th>Total Number of Potential Warehouses (W)</th>
<th>Total Number of Retailers (R)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PB1</td>
<td>5</td>
<td>5</td>
<td>15</td>
</tr>
<tr>
<td>PB2</td>
<td>5</td>
<td>5</td>
<td>20</td>
</tr>
<tr>
<td>PB3</td>
<td>5</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>PB4</td>
<td>7</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>PB5</td>
<td>7</td>
<td>15</td>
<td>20</td>
</tr>
<tr>
<td>PB6</td>
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<td>15</td>
<td>40</td>
</tr>
<tr>
<td>PB7</td>
<td>10</td>
<td>15</td>
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</tr>
<tr>
<td>PB8</td>
<td>10</td>
<td>15</td>
<td>40</td>
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<tr>
<td>PB9</td>
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<td>20</td>
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</tr>
<tr>
<td>PB10</td>
<td>10</td>
<td>20</td>
<td>60</td>
</tr>
</tbody>
</table>

3.5.2 Numerical Experiments (I)

The goal of the first set of numerical analysis is to compare the computational performance of the OA based solution method for solving model P2 with that of traditional
solvers. We set the base case parameters $\mu_0 = 10$, $\sigma_0^2 = 6$, $c_0 = 4$, $f_0 = 300$, $h_0 = 1000$, $A_0 = 10$, $\theta = 0.5$, $z_{\alpha} = 1.65$, and used BARON to solve model P1 and CPLEX 12 to solve the mixed integer conic quadratic model P2. We should note that $z_{\alpha} = 1.65$ corresponds to 95% service level in the supply chain network design and we assume the total number of working days $\eta = 250$. Table 4 shows the computational times and objective function values obtained from the three methods for the first four problem instances. In order to be able to compare the quality of the solutions, BARON, CPLEX and the OA algorithm are solved to optimality gap of zero. An upper limit of 1000 seconds has been set for all the three methods, at which the algorithm would stop and the achieved solutions are reported. As mentioned earlier, the OA master problem does not need to be solved to optimality (Fletcher and Leyffer 1994) and thus it has been solved as soon as a new integer assignment has been found. The reported results clearly demonstrate the inefficiency of solvers in handling models P1 and P2 directly. For instance, we can observe that BARON and CPLEX are inefficient in handling mixed integer nonlinear model P1 and mixed integer conic quadratic model P2 after problem instances Pb3 and Pb4 respectively; whereas OA algorithm is able to efficiently solve these problem instances in reasonable amount of time to optimality.

**Table 4 Comparisons of BARON and Conic solver with the OA algorithm; computation times reported in seconds.**

<table>
<thead>
<tr>
<th>Problem Instance</th>
<th>BARON (Model P1)</th>
<th>Conic (Model P2)</th>
<th>OA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Time</td>
<td>Objective Value</td>
<td>Time</td>
</tr>
<tr>
<td></td>
<td>Objective Value</td>
<td>(Gap%)</td>
<td>Objective Value</td>
</tr>
<tr>
<td>PB1</td>
<td>11.54</td>
<td>1466005.33(0)</td>
<td>0.67</td>
</tr>
<tr>
<td>PB2</td>
<td>11.730</td>
<td>1421320.75(0)</td>
<td>0.842</td>
</tr>
<tr>
<td>PB3</td>
<td>&gt;1000</td>
<td>3872867.69(1.61)</td>
<td>&gt;1000</td>
</tr>
<tr>
<td>PB4</td>
<td>&gt;1000</td>
<td>2792700.759(6.15)</td>
<td>&gt;1000</td>
</tr>
</tbody>
</table>
3.5.3 Numerical Experiments (II)

The goal of the second part of the numerical experiments is to examine the benefits of incorporating demand correlation within the location-inventory problems. The effect of demand correlation in this type of problems has been demonstrated by calculating the savings achieved in the total cost of the network with and without the correlation assumption in the retailer demand. In particular, we first calculate the total cost of the network assuming no retailer's demand correlation and then the same problem instance with the correlation assumption is solved. Savings in the total cost can be calculated by taking the difference of the objective function of the correlated model under the uncorrelated and correlated solutions respectively. Mathematically, the percentage of savings in total cost of the system can be calculated by the following equation:

\[
Savings(\%) := 100 \times \frac{Obj_{Correlated}(Uncorrelated \ Solution) - Obj_{Correlated}}{Obj_{Correlated}(Uncorrelated \ Solution)}
\]

In this test experiment both correlated and the uncorrelated models have been solved by the OA algorithm. Analogous to the previous test, the base parameters are set as \( \mu_0 = 10, \sigma^2_0 = 6, c_0 = 4, f_0 = 300, h_0 = 1000, \theta = 0.5, z_\alpha = 1.65 \) and throughout the study all the model parameters are randomly generated based on the Table 1 if stated otherwise. Each problem instance is solved for twenty five randomly generated model parameters. The OA framework is solved to 1% relative optimality gap (the relative difference between the achieved upper bound and lower bound) and we set a maximum of 200 iterations for the algorithm.
3.5.3.1 Effects of Correlation Coefficient

The first part of the numerical experiments deals with the effect of demand correlation on the total savings achieved across all the networks. In particular, in this test experiment the total savings for the entire networks as a result of changing the correlation coefficient from 0.1 to 0.9 has been calculated. The results of this test are presented in Table 5 where the average and maximum of the savings achieved for all the test networks are reported. The minimum savings achieved for all the test instances is zero, which indicates that both correlated and uncorrelated models provide the same solution for certain instances. From the reported results we can conclude that considering demand correlation in location-inventory problems can lead to the savings in the total cost of the system compared to the network design problem in which correlation in retailers demand is ignored. Also, the results show an increasing pattern in the achieved savings with increase in the correlation coefficient. For instance, the average savings in the total system cost for network PB10 with demand correlation coefficient equals to 0.1 is 3.20% whereas the average savings for correlation coefficient equals to 0.9 would increase to 4.69%. Also the maximum savings in many of network instances is noticeably high. For example in problem PB10 considering the correlation coefficient 0.5 the cost of the uncorrelated model in the worst case can be 18.76% more than the cost of the correlated model which seems to be significant. The maximum of the achieved savings tends to be even more for smaller networks such as PB1 and PB2.
Table 5 Achieved Savings vs. Correlation Coefficient

<table>
<thead>
<tr>
<th></th>
<th>Correlation Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.1</td>
</tr>
<tr>
<td>PB1</td>
<td>Ave</td>
</tr>
<tr>
<td></td>
<td>Max</td>
</tr>
<tr>
<td>PB2</td>
<td>Ave</td>
</tr>
<tr>
<td></td>
<td>Max</td>
</tr>
<tr>
<td>PB3</td>
<td>Ave</td>
</tr>
<tr>
<td>PB4</td>
<td>Ave</td>
</tr>
<tr>
<td></td>
<td>Max</td>
</tr>
<tr>
<td>PB5</td>
<td>Ave</td>
</tr>
<tr>
<td>PB6</td>
<td>Ave</td>
</tr>
<tr>
<td>PB7</td>
<td>Ave</td>
</tr>
<tr>
<td>PB8</td>
<td>Ave</td>
</tr>
<tr>
<td></td>
<td>Max</td>
</tr>
<tr>
<td>PB9</td>
<td>Ave</td>
</tr>
<tr>
<td>PB10</td>
<td>Ave</td>
</tr>
<tr>
<td></td>
<td>Max</td>
</tr>
</tbody>
</table>

Table 6 Location Analysis for Problem Instance PB6

<table>
<thead>
<tr>
<th>Correl</th>
<th>Plant Location</th>
<th>Savings</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>L1-L2-L4-L6-L8-L12</td>
<td>P5</td>
</tr>
<tr>
<td>0.1</td>
<td>L2-L6-L11-L12</td>
<td>P5</td>
</tr>
</tbody>
</table>
We also analyzed the effect of demand correlation on the choice of facilities as warehouse and plant. We presented the results for two representative networks of PB6 and PB8 where the locations of facilities with respect to various correlation coefficients are presented. The results presented in Tables 6 and 7 confirm that the location of the selected facilities for warehouse and plant are different from the locations selected from the uncorrelated problem. For example for problem PB6 when there is no correlation assumption facilities \{L1-L2-L4-L6-L8-L12\} are selected, whereas for correlation coefficients 0.1 to 0.7 the locations of facilities considered as the warehouse are different-\{L2-L6-L11-L12\}. Even for the two cases of correlation coefficient equals to 0.5 and 0.6 the location of the plant is different from the
uncorrelated model. We also observe that the savings achieved for this instance tend to be stable after the correlation coefficient 0.2 which can be due to the fact that the same facilities were selected for correlation coefficient of 0.2 to 0.7. However, this is just for one specific instance and in general as showed in Table 4 the average savings would increase with correlation factor. Similar results are also achieved for network problem PB8 where the locations of correlated and uncorrelated models are different. However, in problem instance PB8 two plants were located to optimally serve the entire supply chain. Also, as we can observe from the results depicted in Table 7 the locations of the plants are subject to change with increase in correlation factor. Furthermore, we should point out that since in this model there is no restriction on plants capacity maximum of two plants are located across all the instances. Finally, with the achieved results we can draw this conclusion that in location-inventory problems the solutions of the correlated and uncorrelated model differ substantially in terms of selected locations for warehouse and plant. In addition, Daskin et al. (2002) and Snyder et al. (2007) showed that increase in safety stock cost parameters would decrease the number of located warehouse facilities. A similar conclusion can be drawn here: an increase in correlation coefficient increases the safety stock cost, and thus decreases the total numbers warehouses.

**Table 7 Location Analysis for Problem Instance PB8**

<table>
<thead>
<tr>
<th>Correlation Coefficient</th>
<th>Warehouse Location</th>
<th>Plant Location</th>
<th>Savings</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>L3-L8-L11-L13-L15</td>
<td>P8,P10</td>
<td>0</td>
</tr>
<tr>
<td>0.1</td>
<td>L4-L8- L13-L15</td>
<td>P8,P10</td>
<td>3.43</td>
</tr>
<tr>
<td>0.2</td>
<td>L4-L8- L13-L15</td>
<td>P8,P10</td>
<td>4.76</td>
</tr>
<tr>
<td>0.3</td>
<td>L4-L8- L13-L15</td>
<td>P8,P10</td>
<td>4.74</td>
</tr>
<tr>
<td>0.4</td>
<td>L4-L8- L13-L15</td>
<td>P8,P10</td>
<td>5.45</td>
</tr>
<tr>
<td>0.5</td>
<td>L4-L8- L13-L15</td>
<td>P8,P10</td>
<td>5.15</td>
</tr>
<tr>
<td>0.6</td>
<td>L3-L4-L7-L13</td>
<td>P6,P10</td>
<td>6.17</td>
</tr>
<tr>
<td>0.7</td>
<td>L3-L4-L7-L13</td>
<td>P6,P10</td>
<td>6.59</td>
</tr>
<tr>
<td>0.8</td>
<td>L3-L4-L7-L13</td>
<td>P6,P10</td>
<td>7.47</td>
</tr>
</tbody>
</table>
The total costs of the inventory with respect to the correlation coefficients for the test instances of PB6 and PB8 are demonstrated in Figures 9 and 10. The illustrated results show that the total inventory costs tend to decrease compared with the case where correlation coefficient equals to zero. The immediate effect of incorporating the correlation coefficient within the location-inventory control problems is the increase in risk pooling effect for every located facility. The increase in risk pooling effect as also mentioned by Daskin et al. (2002) and Snyder et al. (2007) would result in less number of facilities which can further be translated to less total inventory costs. In other words, the more the risk pooling cost the less the tendency to open more facilities. Thus, the total cost associated with inventories decreases with increase in correlation factor.
3.5.3.2 Effect of capacity on savings

The goal of this part of the numerical experiment is to assess the effect of the warehouse capacity level on the savings achieved across all the networks. More specifically we tested the OA algorithm and calculated the savings based on the five capacity levels of \{2,4,6,8,10\}. Also, the correlation coefficient is set at 0.5. Table 8 presents the maximum and the average of the savings across the entire test networks for different values of capacity level. Similar to the previous test the minimum of the achieved savings across all the networks was found to be zero. As we can see from the results the savings achieved for each network is highly coupled with the capacity levels. The general trend across the results is that with increase in the capacity level the savings tend to increase. The increase in savings with corresponding increase in capacity can be translated to the fact that theoretically at higher capacities the feasible region of the mathematical formulation increases which can then lead to the solutions with lower costs and, and also more savings. In other words, at higher capacity levels, there are potentially more feasible combination of locations of facilities and allocation of retailers with less total supply chain cost which finally results in more savings.
### Table 8 Effects of Capacity of Savings

<table>
<thead>
<tr>
<th></th>
<th>Ave</th>
<th>Max</th>
<th>Ave</th>
<th>Max</th>
<th>Ave</th>
<th>Max</th>
<th>Ave</th>
<th>Max</th>
<th>Ave</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>PB1</td>
<td>5.48</td>
<td>34.71</td>
<td>6.31</td>
<td>33.01</td>
<td>7.19</td>
<td>37.37</td>
<td>7.45</td>
<td>37.37</td>
<td>7.64</td>
<td>30.98</td>
</tr>
<tr>
<td>PB2</td>
<td>5.50</td>
<td>27.07</td>
<td>5.59</td>
<td>30.99</td>
<td>5.90</td>
<td>25.19</td>
<td>6.28</td>
<td>29.58</td>
<td>6.63</td>
<td>26.49</td>
</tr>
<tr>
<td>PB3</td>
<td>5.14</td>
<td>26.28</td>
<td>5.37</td>
<td>26.42</td>
<td>5.70</td>
<td>18.91</td>
<td>4.95</td>
<td>31.50</td>
<td>5.35</td>
<td>30.50</td>
</tr>
<tr>
<td>PB5</td>
<td>3.45</td>
<td>20.52</td>
<td>3.98</td>
<td>26.71</td>
<td>4.45</td>
<td>25.72</td>
<td>4.67</td>
<td>25.75</td>
<td>5.10</td>
<td>27.07</td>
</tr>
<tr>
<td>PB6</td>
<td>3.34</td>
<td>15.94</td>
<td>4.17</td>
<td>30.76</td>
<td>4.86</td>
<td>29.97</td>
<td>5.15</td>
<td>24.17</td>
<td>6.59</td>
<td>32.58</td>
</tr>
<tr>
<td>PB7</td>
<td>4.28</td>
<td>17.35</td>
<td>4.64</td>
<td>23.68</td>
<td>5.67</td>
<td>24.71</td>
<td>5.81</td>
<td>24.36</td>
<td>6.58</td>
<td>32.14</td>
</tr>
<tr>
<td>PB8</td>
<td>3.35</td>
<td>13.64</td>
<td>4.82</td>
<td>24.65</td>
<td>5.33</td>
<td>26.01</td>
<td>6.31</td>
<td>26.49</td>
<td>6.56</td>
<td>31.76</td>
</tr>
<tr>
<td>PB9</td>
<td>3.79</td>
<td>16.12</td>
<td>4.86</td>
<td>30.71</td>
<td>5.63</td>
<td>29.95</td>
<td>5.79</td>
<td>30.71</td>
<td>7.74</td>
<td>26.82</td>
</tr>
<tr>
<td>PB10</td>
<td>4.46</td>
<td>16.15</td>
<td>4.99</td>
<td>31.64</td>
<td>6.03</td>
<td>32.38</td>
<td>7.38</td>
<td>36.05</td>
<td>7.54</td>
<td>34.02</td>
</tr>
</tbody>
</table>

#### 3.5.3.3 Effect of holding cost

This part of the numerical experiment is designed to examine the effect of holding cost on the savings achieved in the total cost of the network as the results of considering correlated model over uncorrelated one. In this particular experiment the correlation coefficient for demand has been fixed as 0.5, the base capacity level is set as 4 and we introduce three levels of $h_0=500, 1000$ and 1500 as the base holding cost. The remaining parameters are set the same as the previous test. The associated results are reported in the form of average and maximum savings in Table 9 for different values of the holding costs. As we can conclude from the results a corresponding increase in the savings has been observed with increase in the holding cost. This is mainly because the safety stock cost tends to be more significant as holding costs increase, and thus accounting for demand correlation in the safety stock cost term would lead in more savings in the total cost.
Table 9 Effects of Holding Cost on Savings

<table>
<thead>
<tr>
<th></th>
<th>Base Holding Cost</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$h_0=500$</td>
<td>$h_0=1000$</td>
<td>$h_0=1500$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ave</td>
<td>Max</td>
<td>Ave</td>
<td>Max</td>
</tr>
<tr>
<td>PB1</td>
<td>1.84</td>
<td>12.72</td>
<td>6.11</td>
<td>28.53</td>
</tr>
<tr>
<td>PB2</td>
<td>4.90</td>
<td>33.36</td>
<td>5.66</td>
<td>35.15</td>
</tr>
<tr>
<td>PB3</td>
<td>3.73</td>
<td>24.49</td>
<td>4.71</td>
<td>18.03</td>
</tr>
<tr>
<td>PB4</td>
<td>3.73</td>
<td>24.62</td>
<td>4.43</td>
<td>26.51</td>
</tr>
<tr>
<td>PB5</td>
<td>2.98</td>
<td>11.85</td>
<td>5.26</td>
<td>21.14</td>
</tr>
<tr>
<td>PB6</td>
<td>3.57</td>
<td>19.72</td>
<td>4.21</td>
<td>23.38</td>
</tr>
<tr>
<td>PB7</td>
<td>3.97</td>
<td>19.89</td>
<td>5.52</td>
<td>25.35</td>
</tr>
<tr>
<td>PB8</td>
<td>3.47</td>
<td>23.63</td>
<td>4.31</td>
<td>25.40</td>
</tr>
<tr>
<td>PB9</td>
<td>3.67</td>
<td>19.27</td>
<td>5.19</td>
<td>24.06</td>
</tr>
<tr>
<td>PB10</td>
<td>3.18</td>
<td>22.44</td>
<td>4.48</td>
<td>21.16</td>
</tr>
</tbody>
</table>

3.5.4 Numerical Experiments (III)

The aim of the third part of the numerical experiments is to study the computational performance of the OA algorithm with respect to the parameters such as warehouse capacity, holding cost and the correlation coefficient. More, particularly in this section we perform a thorough sensitivity analysis of the computational time based on these parameters in order to present the impact of these parameters in changing the solution time of the OA algorithm. In this section we present minimum, maximum and the average of the solution time as well as the minimum and maximum of the iterations required by the algorithm to reach 1% of the optimality gap. Similarly we considered the base parameters as $\mu_0 = 10, \sigma_0^2 = 6, c_0 = 4, f_0 = 300, h_0 = 1000, \theta = 0.5, z_{\alpha} = 1.64$ and generated the network parameters based on the Table 1 for twenty five random instances. Before starting this section we first provide computational performance of
base case in order to be able to compare the effect of changes in other test parameters with a base case. The computational performance of the base model in terms of the computational time and number of iterations is depicted in Table 10.

As expected from the results the computational time increases with increase in network size since problems with higher number of suppliers, warehouses, and retailers tend to be harder to solve. From the results we can also conclude that number of retailers has a great impact on the solution time. For example for problem PB9 and PB10 which share the same number of plant and warehouses, increase in number of retailers from 40 to 60 increases the average solution time from 102.31 seconds to 167.76 seconds. Similar pattern exists between problem instances of PB7 and PB6. This can be mostly attributed to the fact that increase in number of retailers would significantly increase the size of the formulation due to the effect of demand correlations among the retailers thus resulting in higher solution time. Nevertheless, number of retailers tends to be less significant in the small size problems with less number of plant and warehouses.

Table 10  Computational Performance of the Base Case

<table>
<thead>
<tr>
<th>Problem Instance</th>
<th>( C_0 = 4 )</th>
<th>Computational Time (sec)</th>
<th>Number of Iteration</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Min</td>
<td>Ave</td>
<td>Max</td>
</tr>
<tr>
<td>PB1</td>
<td>0.14</td>
<td>0.61</td>
<td>1.45</td>
</tr>
<tr>
<td>PB2</td>
<td>0.19</td>
<td>0.62</td>
<td>1.50</td>
</tr>
<tr>
<td>PB3</td>
<td>0.47</td>
<td>2.80</td>
<td>8.01</td>
</tr>
<tr>
<td>PB4</td>
<td>0.92</td>
<td>3.08</td>
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<td>PB5</td>
<td>10.28</td>
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<td>PB7</td>
<td>16.43</td>
<td>56.26</td>
<td>101.95</td>
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<tr>
<td>PB8</td>
<td>26.50</td>
<td>77.75</td>
<td>123.15</td>
</tr>
<tr>
<td>PB9</td>
<td>27.88</td>
<td>102.31</td>
<td>227.63</td>
</tr>
<tr>
<td>PB10</td>
<td>49.36</td>
<td>167.78</td>
<td>396.12</td>
</tr>
</tbody>
</table>
3.5.4.1 Effect of warehouse capacity

In this part, the performance of the proposed OA approach is examined by changing various supply chain related parameters such as demand correlation coefficient and warehouse capacity. Atamtürk et al. (2012) showed that capacity can be an important factor affecting the solution time for a two level location-inventory problem. Therefore in this experiment a sensitivity analysis has been performed based on different values for the base case capacity, defined in Table 1. We keep all the other model parameters in the same setting as the base case and solved OA within 1% of relative optimality gap. To this end we considered four different scenarios of $C_0 = 2$, $C_0 = 6$, $C_0 = 8$, and $C_0 = 10$ for the base capacity level and tested the OA performance according to such capacities. The computational performances of the algorithm for the base capacity levels are reported in Tables 11 and 12. The achieved results highlight that the solution time tends to increase with decrease in capacity level. In other words the tighter the warehouse capacity, more computational effort is required to solve the problem. The increase in computational time is more significant in large size problem instances such as PB8, PB9, and PB10. Furthermore number of iterations required for the algorithm to be completed is more in the case of tight capacity level.
We also demonstrated the number of located warehouse for problem instance PB10 in two cases of no correlation and correlation coefficient equal to 0.5 in Figure 11. The results show that with increase in warehouse capacity the number of located facilities would decrease. This follows from the fact that with high capacity level in warehouses opening a new facility is not economically justified and problem will locate less number of facilities. Also, from the results we can observe that for capacity levels 2, 4, 6, and 8 the number of located warehouses for the correlated case tends to be less compared to the case with no correlation assumption. This can be explained based on the fact that the increase in the effect safety stock cost as a result of considering demand correlation would decrease the number of located warehouses. However when the capacity is so loose the number of located warehoused seems to be the same.

### Table 11 Sensitivity analysis of capacity for base capacity level of $C_0=2$ and $C_0=6$

<table>
<thead>
<tr>
<th>Problem Instance</th>
<th>$C_0=2$</th>
<th></th>
<th>$C_0=6$</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Computational Time (sec)</td>
<td>Number of Iterations</td>
<td>Computational Time (sec)</td>
<td>Number of Iterations</td>
</tr>
<tr>
<td></td>
<td>Min</td>
<td>Ave</td>
<td>Max</td>
<td>Min</td>
</tr>
<tr>
<td>PB1</td>
<td>0.09</td>
<td>0.79</td>
<td>4.70</td>
<td>1</td>
</tr>
<tr>
<td>PB2</td>
<td>0.14</td>
<td>0.70</td>
<td>2.67</td>
<td>2</td>
</tr>
<tr>
<td>PB3</td>
<td>0.78</td>
<td>3.41</td>
<td>8.83</td>
<td>2</td>
</tr>
<tr>
<td>PB4</td>
<td>0.78</td>
<td>3.81</td>
<td>18.63</td>
<td>2</td>
</tr>
<tr>
<td>PB5</td>
<td>10.03</td>
<td>31.30</td>
<td>120.39</td>
<td>1</td>
</tr>
<tr>
<td>PB6</td>
<td>10.94</td>
<td>61.38</td>
<td>131.70</td>
<td>2</td>
</tr>
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<td>PB7</td>
<td>26.11</td>
<td>76.16</td>
<td>144.99</td>
<td>2</td>
</tr>
<tr>
<td>PB8</td>
<td>29.56</td>
<td>94.80</td>
<td>161.92</td>
<td>2</td>
</tr>
<tr>
<td>PB9</td>
<td>32.82</td>
<td>133.09</td>
<td>331.42</td>
<td>5</td>
</tr>
<tr>
<td>PB10</td>
<td>51.69</td>
<td>217.67</td>
<td>810.17</td>
<td>6</td>
</tr>
</tbody>
</table>
Table 12 Sensitivity analysis of capacity for base capacity level of $c_0=8$ and $c_0=10$

<table>
<thead>
<tr>
<th>Problem Instance</th>
<th>$C_0=8$</th>
<th>$C_0=10$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Computational Time (sec)</td>
<td>Number of Iterations</td>
</tr>
<tr>
<td></td>
<td>Min</td>
<td>Ave</td>
</tr>
<tr>
<td>PB1</td>
<td>0.13</td>
<td>0.55</td>
</tr>
<tr>
<td>PB2</td>
<td>0.11</td>
<td>0.59</td>
</tr>
<tr>
<td>PB3</td>
<td>0.23</td>
<td>2.29</td>
</tr>
<tr>
<td>PB4</td>
<td>0.34</td>
<td>3.88</td>
</tr>
<tr>
<td>PB5</td>
<td>3.85</td>
<td>14.75</td>
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<tr>
<td>PB6</td>
<td>10.20</td>
<td>18.13</td>
</tr>
<tr>
<td>PB7</td>
<td>12.45</td>
<td>36.80</td>
</tr>
<tr>
<td>PB8</td>
<td>11.28</td>
<td>56.82</td>
</tr>
<tr>
<td>PB9</td>
<td>17.04</td>
<td>76.65</td>
</tr>
<tr>
<td>PB10</td>
<td>23.51</td>
<td>88.99</td>
</tr>
</tbody>
</table>

3.5.4.2 Effect of holding cost on the solution time

The aim of this part of the numerical experiment is to focus on the effect of holding cost on the solution time of the OA framework in handling the location-inventory problems. In this part, three levels of $h_0=500, 1000$ and $1500$ are considered in order to study the effect of holding cost on the computational effort of the OA algorithm. Other related network parameters are kept as in the base case and the algorithm is solved within 1% of the optimality gap. The results of this test experiment for the entire test problems are presented in Tables 13 and 14 in the form of the computational time and number of iterations. The results clearly show the computational time is highly related to the holding cost level as increase in the holding cost from 500 to 1500 would significantly increase the solution time of the algorithm. This increase tends to be more significant in larger networks such as PB7, PB8, PB9 and PB10. Similarly, the number of
iterations in order to complete the algorithm would increase with the holding cost. The increase in the computational time of the algorithm as a result of increase in the holding cost can be interpreted due to the fact that increase in holding cost increases the cost of associated inventory and safety stock costs which are the nonlinear components of the problem and thus makes the problem harder to solve.

**Table 13 Sensitivity analysis of holding cost for** \( h_0 = 500, 1000 \) and \( 1500 \)

<table>
<thead>
<tr>
<th>Problem Instance</th>
<th>( h_0 = 500 )</th>
<th>( h_0 = 1000 )</th>
<th>( h_0 = 1500 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Computational Time (sec)</td>
<td>Computational Time (sec)</td>
<td>Computational Time (sec)</td>
</tr>
<tr>
<td></td>
<td>Min</td>
<td>Ave</td>
<td>Max</td>
</tr>
<tr>
<td>PB1</td>
<td>0.17</td>
<td>0.66</td>
<td>2.12</td>
</tr>
<tr>
<td>PB2</td>
<td>0.11</td>
<td>0.98</td>
<td>2.09</td>
</tr>
<tr>
<td>PB3</td>
<td>0.17</td>
<td>3.55</td>
<td>4.93</td>
</tr>
<tr>
<td>PB4</td>
<td>0.98</td>
<td>5.12</td>
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</tr>
<tr>
<td>PB5</td>
<td>7.47</td>
<td>14.17</td>
<td>42.54</td>
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<td>PB6</td>
<td>8.61</td>
<td>31.90</td>
<td>95.52</td>
</tr>
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<td>PB7</td>
<td>12.41</td>
<td>42.54</td>
<td>177.95</td>
</tr>
<tr>
<td>PB8</td>
<td>19.16</td>
<td>46.52</td>
<td>277.25</td>
</tr>
<tr>
<td>PB9</td>
<td>21.70</td>
<td>76.23</td>
<td>340.31</td>
</tr>
<tr>
<td>PB10</td>
<td>42.76</td>
<td>93.10</td>
<td>431.45</td>
</tr>
</tbody>
</table>

**Table 14 Number of Iterations for holding cost for** \( h = 500, 1000 \) and \( 1500 \)

<table>
<thead>
<tr>
<th>Problem Instance</th>
<th>( h_0 = 500 )</th>
<th>( h_0 = 1000 )</th>
<th>( h_0 = 1500 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Iterations</td>
<td>Iterations</td>
<td>Iterations</td>
</tr>
<tr>
<td></td>
<td>Min</td>
<td>Max</td>
<td>Min</td>
</tr>
<tr>
<td>PB1</td>
<td>1</td>
<td>9</td>
<td>1</td>
</tr>
<tr>
<td>PB2</td>
<td>2</td>
<td>22</td>
<td>3</td>
</tr>
<tr>
<td>PB3</td>
<td>2</td>
<td>22</td>
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<tr>
<td>PB4</td>
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<td>PB5</td>
<td>2</td>
<td>22</td>
<td>3</td>
</tr>
<tr>
<td>PB6</td>
<td>2</td>
<td>24</td>
<td>3</td>
</tr>
<tr>
<td>PB7</td>
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<td>26</td>
<td>3</td>
</tr>
<tr>
<td>PB9</td>
<td>3</td>
<td>26</td>
<td>3</td>
</tr>
<tr>
<td>PB10</td>
<td>3</td>
<td>28</td>
<td>3</td>
</tr>
</tbody>
</table>
3.5.4.3 Effect of demand correlation coefficient

This section investigates the effects of correlation among the retailers’ demand on computational times. We considered the input parameters in the same setting as in the first experiment and tested the results based on various correlation coefficients. Similar to the previous experiments we considered 1% as the desired level of relative optimality gap for OA. The objective of this test experiment is to investigate effect of considering correlation in retailers demand on the computational performance of the OA algorithm. Toward this goal we tested the computational performance of the OA framework for all test instances while assuming four different values of the retailers demand correlation coefficient 0.0, 0.1, 0.5, and 0.9. Each
problem instance parameters has been randomly generated twenty five times according to Table 1 and the average of the computational times for the considered correlation coefficients has been presented in Figure 12. From the provided results we can observe that the computational effort remains relatively stable across the test instances with increase in correlation coefficients. The stability of the running time in the OA approach with respect to the correlation coefficient is mostly due to the nature of the algorithm which is to solve a sequence of linear integer programs, while in other nonlinear programming methods such as conic programming the weight of the conic terms in the formulation may have an increasing effect on the solution time (see, e.g., Atamtürk et al. 2012).
3.6 Conclusion

This chapter studied a three-level location-inventory problem while assuming demand correlation among the retailers. The proposed model minimizes the total cost of three types of decisions: (i) A multi-level facility location problem to determine the number and location of plants and warehouses, (ii) allocation problem to determine the best assignment of retailers to located warehouses and located warehouses to located plants, and (iii) inventory control decisions at each located warehouse. We formulated the model as a binary integer nonlinear program which was transformed to a mixed integer conic quadratic program. In addition, we presented an outer approximation based algorithm and demonstrated the algorithmic efficiency of such framework for this class of programs. An extensive numerical experiment was conducted to achieve three goals: first to show the efficiency of the OA approach with respect to the commercial optimization solvers such as BARON and CPLEX in handling mixed integer nonlinear and mixed integer conic programs. Secondly, to demonstrate the value of considering correlation by studying the sub-optimality of the solutions obtained by neglecting the impact of correlations. Thirdly, to investigate the performance of the OA framework with respect to increasing size networks and also to provide insights regarding the effect of parameters such as correlation coefficient and warehouses capacity on the total solution time of the algorithm.

There are various avenues of research for extending the current work. One possible extension could be modeling the inventory control decision of retailers and plants. Efficiency of the outer approximation framework can also be improved by intelligently solving the master problem through a heuristic procedure.
4.0 Dispatching trucks for drayage operations

4.1 Introduction

Truck-rail intermodal transport experienced rapid development since the 1980s when the double-stack rail-cars were first introduced in the USA. The double-stack rail-cars significantly reduced the rail-haul costs and made intermodal transport competitive at distances of about 500 miles, whereas previously it could compete with trucks only at distances greater than 750 miles. However, the cost of the highway portion of truck-rail intermodal transport called drayage remained relatively high (Resor and Blaze, 2004). This paper aims to reduce the cost of drayage operations, which is crucial in making the truck-rail intermodal transport more competitive on the market. The contribution of this paper lies in developing a novel model for dispatching trucks that takes into consideration constraints and sources of stochasticity that arise in the real-world applications. The following two paragraphs introduce the relevant aspects of truck-rail intermodal that must be considered in modeling drayage operations.

The typical concept of intermodal truck-rail service is as follows. A tractor with an empty trailer or container is dispatched from the intermodal terminal to a shipper’s location in order to pick up a load. The tractor and driver wait with the trailer/container while it is being loaded and then, in a first drayage operation, transport it to the intermodal terminal (Morlock and Spasović, 1995). If the truck arrives slightly before the train departure, it is directed to a queue associated with the train loading process and the freight is directly loaded on the train (Rizzoli et al., 2002).
In this situation, only one crane operation is required to directly transfer the load from truck to rail car. Otherwise, the truck is directed to the storage area and at least two crane moves are needed to unload the truck and later load it on a rail car. After the freight is hauled to the destination terminal, a second drayage operation delivers the freight to its final destination. After unloading, an empty trailer/container is returned to a pool of empty trailers/containers at the intermodal terminal.

Drayage operations are constrained by several factors. Besides the capacities of trucks, trains and terminal storage facilities, the number of trucks accessing the terminal within a time slot may also be limited. Unscheduled arrivals at a terminal may cause potential problems to both terminal and drayage operations. Terminals would have their resources idle during the off-peak periods, whereas drivers would experience unnecessary waiting time if they arrive during the peak periods. In addition, excessive truck queuing and idling leads to higher diesel engine emissions, a major environmental problem, especially for terminals in large metropolitan areas. In response to growing truck congestion problems both within and outside terminal gates, many US port terminal operators deploy access control systems limiting the total number of appointments available within each time window (Namboothiri and Erera, 2008).

This chapter aims to reduce drayage cost by optimizing truck departure times. It develops a comprehensive probabilistic model that includes the afore-mentioned constraints and different sources of uncertainty that arise in the actual applications (i.e. probabilistic durations of transportation operations and train departure times). This model is general as it makes few assumptions about the applicable distributions. Moreover, it can be applied to optimizing drayage operations for different types of intermodal terminals and truckloads (containers and trailers).
The remainder of this chapter is organized in seven sections. Section 4.2 defines the problem and states assumptions and types of costs built into the model. Section 4.3 reviews related literature on drayage and truck-train intermodal transport and emphasizes the contribution of this paper. Sections 4.4 and 4.5 compute the costs that are built into the mathematical formulation provided in Section 4.6. The numerical results are presented in Section 4.7. Finally, conclusions are drawn and potential extensions of this work proposed in Section 4.8.

4.2 Problem definition, assumptions, and costs

The problem addressed in this research is now presented with an example of truck-train intermodal using trailers. Consider a single drayage firm operating a truck fleet serving an intermodal terminal and a set of surrounding shipper locations in an export-oriented region. The firm attempts to serve a set of requests to move trailers to the terminal with its available fleet. Trucks are assigned consecutive roundtrips whose durations are randomly distributed. Upon a truck’s arrival at the terminal, the trailer is unloaded at a storage facility where it waits to be loaded to the connecting train. If the truck arrives slightly before the train departure, it can unload its trailer directly on the train; this requires fewer crane moves and therefore lower terminal operation cost. In case the trailer does not connect to the designated train, it has to wait for the next connecting train with available capacity.

The trains may not necessarily depart according to schedule and delays may occur due to various reasons, such as late arrivals of the locomotive(s) or delays in other operations or trains. Therefore the train departure is assumed to be randomly distributed over an interval. The duration of the interval may vary from country to country and in some applications it may be a point (i.e. train departs on time). However, we wish to consider the most general case and allow
a potential user of our model to input these intervals or exact departure times based on actual conditions and experience.

Our objective is to develop a model that optimizes truck departure times given the following assumptions:

1. The number of roundtrips that a truck is assigned, as well as their sequence and randomly distributed durations, which include driving and loading/unloading times, are all given.
2. Durations of truck roundtrips are independent.
3. Departures of trains are randomly distributed over the non-overlapping intervals and independent (i.e. the first departure may be uniformly distributed from 12:00-12:15pm and independent from the second one distributed from 3:30-3:50pm). Their distributions can be determined based on historical data.
4. The expected number of trailers in the terminal must never exceed the preset coefficient of a terminal’s dedicated storage capacity (e.g. 0.8 of the dedicated storage capacity).
5. During each time slot, the drayage firm is limited to a maximum number of truck entries to the terminal. Therefore the expected number of arrivals within a time slot shall not exceed the slot capacity (or a multiple of this capacity).
6. There are enough trailers at the terminal.
7. Trucks carry one 40 ft trailer per roundtrip. This assumption can be relaxed to consider possibility of transporting different types or number of trailers if more complex notation is introduced.

We wish to optimize the truck departures while minimizing the overall system cost. In calculating total cost the following are considered:
1. Storage cost, which refers to the cost of storing trailers in terminal’s storage facilities while waiting for a connection.

2. Penalty for late delivery, reflecting the decrease in the freight’s value as delivery is delayed. The penalty associated with a trailer depends on the departure time of a train on which the trailer is loaded.

3. Cost of in-terminal operations, which includes the cost of unloading and loading trailers. The cost of in-terminal operations is lower in cases where trucks arrive slightly before train departures and unload directly on trains.

To find the truck schedule that minimizes the overall system cost, the total cost is formulated as a function of truck departure times. After reviewing related research in Section 4.3, the types of costs listed above are computed in Sections 4.4-4.5 and included in the mathematical formulation of the problem presented in Section 4.6.

4.3 Related literature and expected contribution

Morlok and Spasović (1995) provide an excellent overview of drayage for truck-rail intermodal service and some of their explanations were already cited in the introduction. Spasović (1990), Morlok and Spasović (1994), and Morlok et al. (1995) model the drayage operations using integer programming and argue that central planning for several drayage companies in one terminal-service area can significantly reduce drayage costs. Nozick and Morlok (1997) develop an integer program for planning medium-term operations for an intermodal truck-rail service. Rizzoli et al. (2002) and Gambardella et al. (2002) use simulation to evaluate different management alternatives and organize transport plans for the dispatching of intermodal transport units, respectively. Routing and scheduling of drayage operations are
addressed in Smilowitz (2006) and Francis et al. (2007). A dynamic extension of routing and scheduling in drayage operations is studied by Zhang et al. (2011). Finally, excellent reviews of different problems related to truck-rail intermodal are found in Bontekoning et al. (2004) and in Macharis and Bontekoning (2004).

The model developed in this chapter is related to the work of Marković (2010) and Marković and Schonfeld (2011), who propose a model for optimizing schedules in a single-hub intermodal freight system. They optimize the schedules on outbound (airline) routes for the given information about the probabilistic arrivals on inbound (truck) routes. That model is based on the random durations of truck roundtrips and the advantages and disadvantages of such an approach are discussed there. In this chapter we develop a model using similar ideas for building a probabilistic model. The probabilistic analysis used to compute different types of cost is enhanced by considering additional stochasticity (i.e. random connection times) and constraints inherent to drayage operations (i.e. limited number of truck entries). Stochastic connection times will be introduced in the current analysis and the shifts in truck roundtrip distributions will be optimized.

4.4 Storage cost

To determine the storage cost we need to estimate the dwell time of trailers in the terminal storage. We separate the in-terminal dwell time into two parts. The first part refers to the dwell time from the moment the trailer reaches a terminal until the first connecting train after its arrival, whether or not that train has enough capacity to take the connecting trailer. We call this primary dwell time and compute its expectation in the following subsection. The second part refers to the dwell time from the departure of the first connecting train until departure of the train
actually carrying this trailer. We call it leftover dwell time and compute its expected value in the second subsection.

4.4.1 Expected primary dwell time $E[PDT]$

In this section we formulate $E[PDT]$ which is a function of the truck departure times we seek to optimize, random duration of truck roundtrips, and randomly distributed train departures. Suppose that truck $k$ is assigned $r_k$ consecutive roundtrips, all starting and ending at the terminal. Let’s denote $X_{k,j}$ the random duration of the $j$-th roundtrip and its probability density function (PDF) $f_{k,j}(x_{k,j})$. Let $Y_{k,j}$ denote a random variable describing the $j$-th truck arrival at the terminal. $Y_{k,j}$ is given with the following sum:

$$Y_{k,j} = X_{k,1} + X_{k,2} + \ldots + X_{k,j-1} + X_{k,j} \quad (42)$$

The PDF of a variable $Y_{k,j}$ is defined as the convolution of PDF’s describing duration of $j$ roundtrips (equation 42). Note that the PDF of the first roundtrip is a function of departure time $d_k$ which we seek to optimize. This $d_k$ represents the shift in the distribution representing the first roundtrip. Moreover, PDFs of all the arrivals are functions of $d_k$ as indicated in equation 43.

$$g_{k,j}(y_{k,j}) = f_{k,1}(x_{k,1},d_k) * f_{k,2}(x_{k,2}) * \ldots * f_{k,j-1}(x_{k,j-1}) * f_{k,j}(x_{k,j}) \quad (43)$$

Let’s assume that a trailer should connect to one of $n' - 1$ trains on route $l$ whose departures are represented with random variables $T_{1}^{l}, T_{2}^{l}, \ldots, T_{n'-1}^{l}$ distributed over intervals
(t^{i_1},t^{i_2}), (t^{i_2},t^{i_3})...,(t^{i_{n-1}},t^{i_n}) \) and defined with PDFs \( h^i(t^{i_1}), h^i(t^{i_2}),...,h^i(t^{i_{n-1}}) \). Let’s further assume that if a truck misses all \( n - 1 \) departures, its trailer must wait for the train departing in interval \( (t^{i_{n-1}},t^{i_n}) \) on the following day and PDF \( h^i(t^{i_n}) \). Finally, let’s suppose that the probability that the truck will arrive at the terminal after time \( t^{i_n} \) is negligible and that the following condition holds: \( d_k \leq t^{i_1} \leq t^{i_{n-1}} \leq t^{i_{n+1}} \), \( \forall i, \forall l, \forall k \).

**Proposition 1.** Let \( t^{i_n} = d_k \) and \( PDT^l_{k,l} \) denote the primary dwell time of the trailer carried by truck \( k \) in the \( j \)-th roundtrip and connecting to route \( l \). The \( E[PDT^l_{k,l}] \) is then computed as follows:

\[
E[PDT^l_{k,l}] = \sum_{i=1}^{d_k} \int_{t^{i-1}}^{t^{i+1}} (t^l_i - y_{k,j}) \cdot g_{k,j}(y_{k,j}) \cdot h^i(t^l_i) \cdot dt^l_i \cdot dy_{k,j} \\
+ \sum_{i=1}^{d_k} \int_{t^{i-1}}^{t^{i+1}} (t^l_i - y_{k,j}) \cdot g_{k,j}(y_{k,j}) \cdot h^i(t^l_i) \cdot dt^l_i \cdot dy_{k,j} \\
+ \sum_{i=1}^{d_k} \int_{t^{i-1}}^{t^{i+1}} (t^l_i - y_{k,j}) \cdot g_{k,j}(y_{k,j}) \cdot h^i(t^l_i) \cdot h^i(t^l_{i+1}) \cdot dt^l_{i+1} \cdot dt^l_i \cdot dy_{k,j} \tag{44}
\]

**Proof.** The proof by induction is provided in the appendix due to lengthiness.

With equation 3 we can compute the expected primary dwell time of trailers connecting to all train routes and being transported by all the trucks in all the roundtrips. This is done by introducing parameter \( p^l_{k,j} \) which equals 1 if the trailer carried by truck \( k \) in the \( j \)-th roundtrip is connecting to train route \( l \) and 0 otherwise. Now we simply sum equation 44 for all the roundtrips, trucks and train routes:
\[ E[PDT] = \sum_{i=1}^{m} \sum_{k=1}^{y} \sum_{j=1}^{x} p_{k,j}^l \cdot E[PDT_{k,j}^l] \]  

(45)

### 4.4.2 Expected leftover dwell time \( E[LDT] \)

Since the above computation does not consider the possibility that a trailer waits longer than period \( (T_i^l - Y_{k,j}) \) due to limited train capacity, the additional calculations are needed. For example, if the expected number of trailers arriving in interval \( (T_{i-1}^l, T_i^l) \) and connecting to route \( l \) exceeds the capacity of the train departing at \( T_i^l \), we must consider the leftover dwell time of leftover trailers. In order to calculate the expected leftover dwell time we must compute the expected number of trailers connecting to route \( l \) and arriving in each of \( (T_{i-1}^l, T_i^l) \) intervals. The aforementioned expectation will be calculated in subsection 3.4.3 based on the expected number of trailers arriving in the \( (0, T_i^l) \) interval which is computed in subsection 3.4.2. Please note that \( 0 \) denotes the beginning of the observed time period. Finally, the last subsection 3.4.4 provides an algorithm which is used to compute the expected leftover dwell time \( E[LDT] \) using these expectations.

### 4.4.3 Expected number of trailers arriving in interval \( (0, T_i^l) \) and connecting to route \( l \)

Let’s suppose again that truck \( k \) is assigned \( r_k \) consecutive roundtrips and that we must calculate the expected number of trailers arriving in interval \( (0, T_i^l) \) and connecting to route \( l \). To do so, we must first find the probability that \( a \) arrivals occur within the \( (0, T_i^l) \) interval. In other words, we need to calculate the probability that the first \( a \) roundtrips end prior to \( T_i^l \), while the subsequent roundtrip ends after \( T_i^l \).
\[ P(a) = P(t_i^l < T_i^l < t_i^w; X_{k,1} < T_i^l; \ldots; X_{k,1} + \ldots + X_{k,a} < T_i^l; X_{k,1} + \ldots + X_{k,a+1} > T_i^l) \] (46)

The aforementioned probability is given with an \((a + 2)\)-dimensional integral:

\[
P(a) = \int_{t_i^l}^{t_i^w} dt_i^l \int_{x_{k,1}}^{t_i^l - x_{k,1}} dx_{k,1} \ldots \int_{x_{k,a+1}}^{t_i^l - x_{k,a+1}} dx_{k,a+1} \int_{x_{k,1} - x_{k,2} \ldots}^{t_i^l - x_{k,a-1} - x_{k,a}} dx_{k,a} \int_{x_{k,1} - x_{k,2} \ldots}^{t_i^l - x_{k,a-1} - x_{k,a}} dx_{k,a+1} w(t_i^l, x_{k,1}, x_{k,2}, \ldots, x_{k,a+1}) \cdot dx_{k,a+1} \] (47)

Note that \(w(t_i^l, x_{k,1}, x_{k,2}, \ldots, x_{k,a+1})\) from the above equation represents the joint probability density function of random variables \(T_i^l, X_{k,1}, X_{k,2}, \ldots, X_{k,a+1}\). Since durations of roundtrips and train departure times are independent here, we can obtain the joint PDF by simply multiplying \((a + 2)\) PDF’s.

\[
w(t_i^l, x_{k,1}, x_{k,2}, \ldots, x_{k,a+1}) = h_i^l(t_i^l) \cdot f_{k,1}(x_{k,1}, d_k) \cdot \prod_{j=2}^{a+1} f_{k,j}(x_{k,j}) \] (48)

After computing the probability of \(a\) arrivals in interval \((0, T_i^l)\), we can calculate the expected number of trailers connecting to route \(l\) that truck \(k\) delivers to the terminal in the aforementioned interval as:

\[
E[TR_i^l(0, T_i^l)] = \sum_{a=1}^{A} P(a) \sum_{j=1}^{V} p_{k,j} \] (49)

Now, we can consider the general case including multiple trucks making multiple roundtrips. For this case, the expected amount of freight connecting to route \(l\) and arriving at the terminal in interval \((0, T_i^l)\) can be obtained by simply summing equation 49 for all \(v\) trucks.
$E[TR^i(0,T_i^i)] = \sum_{k=1}^{i} E[TR_k^i(0,T_i^i)] \quad (50)$

### 4.4.4 Expected number of trailers arriving in interval $(T_{i-1}^i, T_i^i)$ and connecting to route $l$

The expected number of trailers connecting to route $l$ and arriving at the terminal in interval $(0, T_i^i)$ was computed above. Based on that result, we are able to calculate the expected number of trailers connecting to route $l$ and arriving at the terminal in interval $(T_{i-1}^i, T_i^i)$, denoted as $E[TR^i(T_{i-1}^i, T_i^i)]$. This $E[TR^i(T_{i-1}^i, T_i^i)]$ equals the expected number of trailers arriving at the terminal in $(0, T_i^i)$ minus the expected number of trailers arriving in $(0, T_{i-1}^i)$.

$$E[TR^i(T_{i-1}^i, T_i^i)] = E[TR^i(0,T_i^i)] - E[TR^i(0,T_{i-1}^i)] \quad (51)$$

Having derived the previous expectation, we are now able to determine the expected number of trailers arriving between consecutive train departures and thereby estimate the leftover dwell time that occurs due to limited train capacity.

### 4.4.5 Algorithm for computing expected leftover dwell time $E[LDT]$  

The algorithm for computing leftover dwell time for trailers connecting to route $l$ uses the previously derived expectation $E[TR^i(T_{i-1}^i, T_i^i)]$. For the given truck departures, it examines the expected number of trailers arriving between consecutive train departures and determines whether this number exceeds the train’s capacity $A_i^l$. If it exceeds $A_i^l$, the algorithm computes associated leftover dwell time and adds it to $E[LDT^i]$.
Let’s denote as \( s_i^l \) the number of trailers connecting to route \( l \) left in storage after the \( i \)-th train departure, and assign initial values of zero to \( s_i^l \) and \( E[LDT^l] \). Now, we can compute the leftover dwell time for the freight connecting to route \( l \) with the recursive relation given in equations 52 through 55.

\[
E[LDT^l] = 0; \quad s_0^l = 0
\]  
\[ (52) \]

\[
For \ i = 1 \ to \ n^l - 1
\]  
\[ (53) \]

\[
s_i^l = \max\{0, s_{i-1}^l + E[TR^l(T_{i-1}^l, T_i^l) - A_i^l]\}
\]  
\[ (54) \]

\[
E[LDT^l] = E[LDT^l] + s_i^l \cdot E[T_{i+1}^l - T_i^l]
\]  
\[ (55) \]

Finally, after computing leftover dwell for freight connecting to route \( l \), we can calculate total leftover dwell time by simply summing expression 55 for all \( m \) train routes.

\[
E[LDT] = \sum_{l=1}^{m} E[LDT^l]
\]  
\[ (56) \]

4.4.6 Computing the storage cost

Since we know how to calculate the expected primary and leftover dwell times, we can compute the expected trailer-hours by summing two expectations. To obtain the storage cost, we multiply the sum of two expectations by the unit storage cost \( C_{DT} \).

\[
SC = (E[PDT] + E[LDT])C_{DT}
\]  
\[ (57) \]
4.5 In-terminal operation and penalty cost

4.5.1 In-terminal operation cost

As previously argued, the cost of in-terminal operations can be reduced when a truck arrives at the terminal slightly prior to the train departure and unloads directly on the train. In equation 10 we formulate the expected number of trailers connecting to \( l \) and arriving to terminal on \((T_{i-1}^l, T_i^l)\), where \(T_{i-1}^l\) and \(T_i^l\) are random variables defined over intervals \((t_{i-1}', t_{i-1}'')\) and \((t_i', t_i'')\), such that \(t_{i-1}' < t_{i-1}'' < t_i'' < t_i'^{''}\). We can use this result to estimate the expected number of trailers unloaded directly on trains. We first denote as \( \Delta t \) the time interval such that a truck arriving within the \((T_i^l - \Delta t, T_i^l)\) will unload directly onto the train departing at \( T_i^l \). If we assume that \( \Delta t > t_i'' - t_i' \), we can use equation 51 to compute the expected number of trailers connecting to route \( l \) that will be transferred directly from truck to train, as follows:

\[
b_{d}^i = \sum_{i=1}^{d} E[TR^i (T_i^l - \Delta t, T_i^l)]
\]  

(58)

To find the total number of trailers loaded directly to trains, we sum equation 58 for all \( m \) train routes:

\[
b_{d} = \sum_{i=1}^{m} \sum_{i=1}^{d} E[TR^i (T_i^l - \Delta t, T_i^l)]
\]  

(59)

The remaining trailers will be processed through the terminal storage facility and another cost will be associated with it. We denote as \( C_{td} \) the unit cost of in-terminal operations for the case when trucks take trailers directly to the train. We denote as \( C_{tr} \) the unit cost of in-terminal
operations when the trailer is processed through storage. Finally, if we denote as $U$ the overall number of trailers, then the total in-terminal operation cost is:

$$ IC = b_d C_{sd} + (U - b_d) C_{tr} $$  \hspace{1cm} (60)

### 4.5.2 Penalty cost

In order to estimate the late delivery penalty, we formulate the time-dependent penalty function $p(T_i^l)$ and assume that train departure time $T_i^l$ is relevant for calculating the penalty. For example, if $z_i^l$ trailers are loaded on the train departing at $T_i^l$, the corresponding penalty will be $z_i^l p(T_i^l)$. Similarly to the algorithm 52-55, we compute the penalty cost using a recursive relation given in equations 61 through 64. We use again the expected number of trailers connecting to $l$ and arriving in the $(T_{i-1}^l, T_i^l)$ interval.

$$ PC^l = 0; \ s_0^l = 0 $$  \hspace{1cm} (61)

$$ For \ i = 1 \ to \ n^l $$  \hspace{1cm} (62)

$$ z_i^l = \min\{A_i^l, s_{i-1}^l + E[TR^l(T_{i-1}^l, T_i^l)]\} $$  \hspace{1cm} (63)

$$ PC^l = PC^l + z_i^l \cdot p(T_i^l) $$  \hspace{1cm} (64)

To calculate the total penalty cost associated with trailers carried at all train routes, we need to sum equation 64 for all $m$ routes:

$$ PC = \sum_{i=1}^{m} PC^l $$  \hspace{1cm} (65)
4.6 Mathematical formulation of the model

The train capacity constraint has already been considered by calculating the expected leftover dwell time $E[LDT]$. However, the constraints regarding terminal storage and slot capacities have not yet been included. Moreover, the truck departure times might be restricted to certain intervals.

We must first ensure that the expected number of trailers never exceeds a coefficient of the dedicated storage capacity. We define a vector $T$ such that its elements represent train departures on all $m$ train routes organized in ascending order. Element $T_i$ represents the $i$-th departure from the terminal. Let $s_i$ denote the total number of trailers left in the terminal after the $i$-th train departure, similarly to $s'_i$ in the algorithm in equations 11 through 14. If we denote as $n$ the total number of train departures from terminal and $E[TR(T_{i-1}, T_i)]$ the expected total number of trailers arriving between two consecutive train departures, the storage constraint is given in equation 66. Please note that $T_0 = 0$, $s_0 = 0$, and that $S_c$ and $m_s$ denote the dedicated storage capacity and storage coefficient, respectively.

$$ s_{i-1} + E[TR(T_{i-1}, T_i)] \leq m_s \cdot S_c, \quad i = 1, \ldots, n \quad (66) $$

Let $t_h$ denote the beginning of the $h$-th time slot within the observed time horizon and let $AC_h$ denote the access capacity within $h$-th time slot. Then the slot capacity constraint is given in equation 67. It should be noted that the boundaries of the interval $(t_{h-1}, t_h)$ are points rather than intervals since $t_{h-1}$ and $t_h$ are not random variables.
General working agreements may allow a trucking company to schedule truck departures only within certain time windows:

$$LB_k \leq d_k \leq UB_k, \quad k = 1, ..., v$$  \hspace{1cm} (68)

We have explained the types of costs and constraints considered. Now we can provide the mathematical formulation of the model in equations 69-72.

$$Min TC = SC + IC + PC$$  \hspace{1cm} (69)

Subject to:

$$s_{i-1} + E[TR(T_{i-1}, T_i)] \leq m_s \cdot S_c, \quad i = 1, ..., n$$  \hspace{1cm} (70)

$$E[TR(t_{h-1}, t_h)] \leq AC_h, \quad h = 1, ..., H$$ \hspace{1cm} (71)

$$LB_k \leq d_k \leq UB_k, \quad k = 1, ..., v$$ \hspace{1cm} (72)

It should be noted that formulation 69-72 is given in compact form and includes all the results from previous sections.

4.7 Numerical results

The developed mathematical model is tested on a simulated case study and sensitivity analyses are provided. A genetic algorithm (GA) (Michalewicz, 1996) is implemented in Matlab on a PC with an AMD Athlon 3300 GHz processor with 6 cores and used to optimize truck departures for drayage operations. Matlab is particularly suitable for this implementation due to
its built-in routines for both symbolic and numeric integrations required to solve multidimensional integrals which are inherent to formulation 28-31. In addition, Matlab allows relatively straightforward parallelization of GA on multiple cores which is an important feature for this computationally demanding optimization. The representation of a chromosome is straightforward, where the $k$-th gene represents the departure time of the $k$-th truck. The population size is set at 12 individuals, and one point crossover and mutation are applied. The GA is run 10 times with different seeds for generating initial populations and the best solution is reported. Finally, since the GA is not guaranteed to find a globally optimal solution, we refer to the resulting schedules as “optimized” rather than “optimal”.

4.7.1 Drayage for a single train route

Here we design and solve a numerical example for the case of a trucking company providing drayage for trailers connecting to the same train route with two scheduled departures during the current day. The train departures are assumed to be uniformly distributed over 30 min intervals with parameters shown in Table 15. We observe 10 trucks each making 3 exponentially distributed roundtrips with means provided in Table 16. Assuming the remaining inputs for capacities, costs and time windows from Table 15, we optimize the truck departure times. It should be noted that we assume hourly time slots for limiting the expected number of truck entries to the terminal.
Table 15 Input data for the case with a single train route

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>dedicated train capacity</td>
<td>$A_{1/2}$</td>
</tr>
<tr>
<td>dedicated storage capacity</td>
<td>$m_x \cdot S_e$</td>
</tr>
<tr>
<td>storage cost</td>
<td>$SC$</td>
</tr>
<tr>
<td>maximum expected truck entries</td>
<td>$AC_h$</td>
</tr>
<tr>
<td>time for direct loading</td>
<td>$\Delta t$</td>
</tr>
<tr>
<td>in-terminal operations</td>
<td>$C_{ir}$</td>
</tr>
<tr>
<td>costs for (in)direct loading</td>
<td>$C_{id}$</td>
</tr>
<tr>
<td>first train departure</td>
<td>$T_1^i$</td>
</tr>
<tr>
<td>second train departure</td>
<td>$T_2^i$</td>
</tr>
<tr>
<td>train departure on the following day</td>
<td>$T_3^i$</td>
</tr>
<tr>
<td>penalty function</td>
<td>$p(t)$</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>16 trailers</td>
</tr>
<tr>
<td></td>
<td>20 trailers</td>
</tr>
<tr>
<td></td>
<td>40 $/$trailer-hr</td>
</tr>
<tr>
<td></td>
<td>2.5-5.5 entries/hr</td>
</tr>
<tr>
<td></td>
<td>1.5 hrs</td>
</tr>
<tr>
<td></td>
<td>70 $/$trailer</td>
</tr>
<tr>
<td></td>
<td>35 $/$trailer</td>
</tr>
<tr>
<td></td>
<td>$T_1^i \sim U(7.4,7.9)$ hr</td>
</tr>
<tr>
<td></td>
<td>$T_2^i \sim U(13.1,13.6)$ hr</td>
</tr>
<tr>
<td></td>
<td>$T_3^i \sim U(24.0,24.5)$ hr</td>
</tr>
<tr>
<td></td>
<td>$0$ $/$trailer if $t \in (7.4,7.9)$</td>
</tr>
<tr>
<td></td>
<td>$100$ $/$trailer if $t \in (13.1,13.6)$</td>
</tr>
<tr>
<td></td>
<td>$400$ $/$trailer if $t \in (24.0,24.5)$</td>
</tr>
</tbody>
</table>
We optimize the system for different limits on the expected number of truck entries to the terminal. The optimized truck dispatching times and corresponding total cost rounded to the nearest dollar are given in Table 17. The problem is infeasible for $AC_h = 2.5$ truck entries/hr and the total cost decreases as this constraint is relaxed to 4.5 truck entries/hr.

<table>
<thead>
<tr>
<th>truck</th>
<th>1st roundtrip [hr]</th>
<th>2nd roundtrip [hr]</th>
<th>3rd roundtrip [hr]</th>
<th>Departure windows $[LB_k, UB_k]$ [hr]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$1/\lambda = 2.2$</td>
<td>$1/\lambda = 1.0$</td>
<td>$1/\lambda = 4.0$</td>
<td>[0,7]</td>
</tr>
<tr>
<td>2</td>
<td>$1/\lambda = 2.8$</td>
<td>$1/\lambda = 2.4$</td>
<td>$1/\lambda = 2.6$</td>
<td>[0,7]</td>
</tr>
<tr>
<td>3</td>
<td>$1/\lambda = 2.4$</td>
<td>$1/\lambda = 1.9$</td>
<td>$1/\lambda = 1.4$</td>
<td>[0,7]</td>
</tr>
<tr>
<td>4</td>
<td>$1/\lambda = 3.2$</td>
<td>$1/\lambda = 3.0$</td>
<td>$1/\lambda = 2.2$</td>
<td>[0,7]</td>
</tr>
<tr>
<td>5</td>
<td>$1/\lambda = 2.3$</td>
<td>$1/\lambda = 3.4$</td>
<td>$1/\lambda = 2.2$</td>
<td>[0,7]</td>
</tr>
<tr>
<td>6</td>
<td>$1/\lambda = 2.0$</td>
<td>$1/\lambda = 2.8$</td>
<td>$1/\lambda = 3.5$</td>
<td>[0,7]</td>
</tr>
<tr>
<td>7</td>
<td>$1/\lambda = 1.9$</td>
<td>$1/\lambda = 3.5$</td>
<td>$1/\lambda = 2.8$</td>
<td>[0,7]</td>
</tr>
<tr>
<td>8</td>
<td>$1/\lambda = 2.5$</td>
<td>$1/\lambda = 2.2$</td>
<td>$1/\lambda = 2.0$</td>
<td>[0,7]</td>
</tr>
<tr>
<td>9</td>
<td>$1/\lambda = 3.4$</td>
<td>$1/\lambda = 1.0$</td>
<td>$1/\lambda = 2.5$</td>
<td>[0,7]</td>
</tr>
<tr>
<td>10</td>
<td>$1/\lambda = 1.2$</td>
<td>$1/\lambda = 1.9$</td>
<td>$1/\lambda = 2.5$</td>
<td>[0,7]</td>
</tr>
<tr>
<td>$AC_h$</td>
<td>tr.1</td>
<td>tr.2</td>
<td>tr.3</td>
<td>tr.4</td>
</tr>
<tr>
<td>-------</td>
<td>------</td>
<td>------</td>
<td>------</td>
<td>------</td>
</tr>
<tr>
<td>2.5</td>
<td>NN</td>
<td>NN</td>
<td>NN</td>
<td>NN</td>
</tr>
<tr>
<td></td>
<td>1.6</td>
<td>2.8</td>
<td>4.7</td>
<td>5.6</td>
</tr>
<tr>
<td>3.0</td>
<td>0</td>
<td>8</td>
<td>9</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>2.7</td>
<td>2.4</td>
<td>4.6</td>
<td>2.4</td>
</tr>
<tr>
<td>3.5</td>
<td>7</td>
<td>5</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>4.0</td>
<td>3.0</td>
<td>3.6</td>
<td>2.1</td>
</tr>
<tr>
<td>4.0</td>
<td>8</td>
<td>5</td>
<td>7</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>4.3</td>
<td>3.1</td>
<td>3.7</td>
<td>2.6</td>
</tr>
<tr>
<td>4.5</td>
<td>9</td>
<td>9</td>
<td>8</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>4.3</td>
<td>3.0</td>
<td>4.0</td>
<td>2.4</td>
</tr>
<tr>
<td>5.0</td>
<td>3</td>
<td>8</td>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>5.5</td>
<td>3</td>
<td>8</td>
<td>2</td>
<td>8</td>
</tr>
</tbody>
</table>

Note: NN – not a number, Inf - infinity
We can observe that the limit on maximum expected truck entries per hour is binding for values 2.5-4 by comparing the first and the last column in Table 17. For values 4.5-5.5 it is not binding, as the total cost and the expected number of truck entries during the most congested (critical) hour remain approximately constant. It is also notable that marginal savings decrease as $AC_h$ is relaxed for as long $AC_h$ is binding. Otherwise, the marginal savings are 0, as expected.

The above results are obtained by running the GA through 300 generations. The computation time is about 16 hours and the GA typically converges within 200 generations. Finally, it should be noted that the above sensitivity analysis results are expected and that we show them here as a verification of our mathematical and computer model. In the following section we apply our model to optimize a case with multiple train routes.

4.7.2 Drayage for multiple train routes

Now we design and solve a numerical example including a trucking company that provides drayage for trailers connecting to two different train routes. Our input represent 14 trucks each making three exponentially distributed roundtrips and triangularly distributed train departures. This time we specify a different distribution for train departures to show the robustness of the model that was developed for a general case without assuming particular distributions. The input data are summarized below.
### Table 18 Input data for the case with multiple train routes

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Formula</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>dedicated train capacity</td>
<td>$A_{1/2}$</td>
<td>16 trailers</td>
</tr>
<tr>
<td>dedicated storage capacity</td>
<td>$m_s \cdot S_c$</td>
<td>15-35 trailers</td>
</tr>
<tr>
<td>storage cost</td>
<td>$SC$</td>
<td>40 $/trailer-hr</td>
</tr>
<tr>
<td>maximum truck entries</td>
<td>$AC_k$</td>
<td>8 entries/hr</td>
</tr>
<tr>
<td>time for direct loading</td>
<td>$\Delta t$</td>
<td>1.5 hrs</td>
</tr>
<tr>
<td>in-terminal operations</td>
<td>$C_{ir}$</td>
<td>70 $/trailer</td>
</tr>
<tr>
<td>costs for (in)direct loading</td>
<td>$C_{sd}$</td>
<td>35 $/trailer</td>
</tr>
<tr>
<td>first train departure on route 1</td>
<td>$T_i^1$</td>
<td>$T_i^1 \sim Tri(8.4,8.4,8.9) \ hr$</td>
</tr>
<tr>
<td>second train departure on route 1</td>
<td>$T_i^2$</td>
<td>$T_i^2 \sim Tri(14.1,14.1,14.6) \ hr$</td>
</tr>
<tr>
<td>train departure on route 1 on the following day</td>
<td>$T_i^3$</td>
<td>$T_i^3 \sim Tri(24.0,24.0,24.5) \ hr$</td>
</tr>
<tr>
<td>first train departure on route 2</td>
<td>$T_i^2$</td>
<td>$T_i^2 \sim Tri(6.4,6.4,6.9) \ hr$</td>
</tr>
<tr>
<td>second train departure on route 2</td>
<td>$T_i^2$</td>
<td>$T_i^2 \sim Tri(11.1,11.1,11.6) \ hr$</td>
</tr>
<tr>
<td>train departure on route 2 on the following day</td>
<td>$T_i^2$</td>
<td>$T_i^2 \sim Tri(26.0,26.0,26.5) \ hr$</td>
</tr>
<tr>
<td>penalty function</td>
<td>$p(t)$</td>
<td>$\begin{cases} 0 \text{$/trailer$ if } t \in (6.4,8.9) \ 100 \text{$/trailer$ if } t \in (11.1,14.6) \ 400 \text{$/trailer$ if } t \in (24.0,26.5) \end{cases}$</td>
</tr>
</tbody>
</table>
Table 19 Exponentially distributed truck roundtrip durations, connecting train route, and departure windows

<table>
<thead>
<tr>
<th>truck</th>
<th>1st roundtrip [hr]</th>
<th>Connecting train route</th>
<th>2nd roundtrip [hr]</th>
<th>Connecting train route</th>
<th>3rd roundtrip [hr]</th>
<th>Connecting train route</th>
<th>Departure windows [hr]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1/λ = 2.2</td>
<td>1</td>
<td>1/λ = 1.0</td>
<td>1</td>
<td>1/λ = 4.0</td>
<td>2</td>
<td>[0,3]</td>
</tr>
<tr>
<td>2</td>
<td>1/λ = 2.8</td>
<td>2</td>
<td>1/λ = 2.4</td>
<td>1</td>
<td>1/λ = 2.6</td>
<td>2</td>
<td>[4,6]</td>
</tr>
<tr>
<td>3</td>
<td>1/λ = 2.4</td>
<td>2</td>
<td>1/λ = 1.9</td>
<td>1</td>
<td>1/λ = 1.4</td>
<td>1</td>
<td>[1,4]</td>
</tr>
<tr>
<td>4</td>
<td>1/λ = 3.2</td>
<td>1</td>
<td>1/λ = 3.0</td>
<td>2</td>
<td>1/λ = 2.2</td>
<td>2</td>
<td>[2,5]</td>
</tr>
<tr>
<td>5</td>
<td>1/λ = 2.3</td>
<td>2</td>
<td>1/λ = 3.4</td>
<td>2</td>
<td>1/λ = 2.2</td>
<td>1</td>
<td>[1,5]</td>
</tr>
<tr>
<td>6</td>
<td>1/λ = 2.0</td>
<td>1</td>
<td>1/λ = 2.8</td>
<td>2</td>
<td>1/λ = 3.5</td>
<td>1</td>
<td>[3,6]</td>
</tr>
<tr>
<td>7</td>
<td>1/λ = 1.9</td>
<td>1</td>
<td>1/λ = 3.5</td>
<td>1</td>
<td>1/λ = 2.8</td>
<td>2</td>
<td>[2,6]</td>
</tr>
<tr>
<td>8</td>
<td>1/λ = 2.5</td>
<td>2</td>
<td>1/λ = 2.2</td>
<td>1</td>
<td>1/λ = 2.0</td>
<td>2</td>
<td>[0,6]</td>
</tr>
<tr>
<td>9</td>
<td>1/λ = 3.4</td>
<td>1</td>
<td>1/λ = 1.0</td>
<td>1</td>
<td>1/λ = 2.5</td>
<td>2</td>
<td>[0,6]</td>
</tr>
<tr>
<td>10</td>
<td>1/λ = 1.2</td>
<td>2</td>
<td>1/λ = 1.9</td>
<td>1</td>
<td>1/λ = 2.5</td>
<td>2</td>
<td>[0,6]</td>
</tr>
<tr>
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<td>1/λ = 2.0</td>
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<td>1/λ = 1.6</td>
<td>2</td>
<td>1/λ = 3.8</td>
<td>1</td>
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<td>2</td>
<td>1/λ = 1.6</td>
<td>1</td>
<td>[0,6]</td>
</tr>
<tr>
<td>13</td>
<td>1/λ = 2.5</td>
<td>2</td>
<td>1/λ = 2.0</td>
<td>2</td>
<td>1/λ = 1.5</td>
<td>1</td>
<td>[0,6]</td>
</tr>
<tr>
<td>14</td>
<td>1/λ = 2.0</td>
<td>1</td>
<td>1/λ = 1.7</td>
<td>2</td>
<td>1/λ = 3.4</td>
<td>1</td>
<td>[0,6]</td>
</tr>
</tbody>
</table>
We optimize the system for different storage capacities. The optimized truck dispatching times and corresponding total cost rounded to the nearest dollar are given in Table 20. The GA is run through 350 generations which require about 30 hours of computation.

<table>
<thead>
<tr>
<th>$m_s \cdot S_c$</th>
<th>Truck dispatching times</th>
<th>TC[$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>[traile r]</td>
<td>tr.1 tr.2 tr.3 tr.4 tr.5 tr.6 tr.7 tr.8 tr.9</td>
<td>tr.1 tr.1 tr.1 tr.1</td>
</tr>
<tr>
<td>35</td>
<td>2.5 4.0 2.6 2.0 1.0 3.0 2.0 1.3 1.3</td>
<td>2.6 2.4 2.3 1.6 2.4</td>
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<tr>
<td></td>
<td>1 0 1 0 0 0 0 6 9 8</td>
<td>5 2 0 0 4</td>
</tr>
<tr>
<td>30</td>
<td>2.4 4.0 2.8 2.0 1.0 3.0 2.0 1.2 1.6</td>
<td>2.7 2.4 2.4 1.7 2.6</td>
</tr>
<tr>
<td></td>
<td>6 0 9 0 0 0 0 0 7</td>
<td>0 0 8 9 4</td>
</tr>
<tr>
<td>25</td>
<td>2.4 4.0 2.8 2.0 1.0 3.0 2.0 1.5 1.7</td>
<td>2.8 2.0 2.2 1.5 2.5</td>
</tr>
<tr>
<td></td>
<td>9 0 6 1 0 0 0 1 4</td>
<td>6 9 0 1 4</td>
</tr>
<tr>
<td>20</td>
<td>3.0 4.0 4.0 2.0 1.4 3.0 4.3 3.1 2.9</td>
<td>3.3 2.7 3.5 1.8 4.1</td>
</tr>
<tr>
<td></td>
<td>0 0 0 0 9 0 2 4 5</td>
<td>5 7 2 2 9</td>
</tr>
<tr>
<td>15</td>
<td>3.0 4.0 3.5 2.5 3.4 5.2 5.6 1.5 6.5</td>
<td>5.9 3.7 5.1 2.1 3.3</td>
</tr>
<tr>
<td></td>
<td>0 0 7 8 9 7 7 2 9</td>
<td>5 7 0 6 4</td>
</tr>
<tr>
<td>10</td>
<td>NN NN NN NN NN NN NN NN NN</td>
<td>NN NN NN NN Inf</td>
</tr>
</tbody>
</table>

Note: NN – not a number, Inf - infinity

From Table 20 it can be observed that the storage constraint is not binding for capacities of 25 to 35 trailers, since the GA converges to approximately the same solutions. Total cost
increases as the storage capacity decreases and the problem becomes infeasible when the storage capacity is set to 10 trailers.

4.8 Conclusions

This paper analyzed truck-train intermodal transport and focused on drayage operations whose cost reduction is crucial for increasing market share of this intermodal service. This work presented an optimization approach aimed towards reducing drayage cost by optimizing dispatching decisions. A comprehensive model for dispatching truck in drayage operations was developed while considering several sources of uncertainty as well as constraints arising in the real-world intermodal operations. The proposed model was tested on two case studies including truck-train trailer transport. Numerical examples showed that the proposed model could cope with different distributions of random variables, which increases its applicability since distributions may vary with applications. Moreover, sensitivity analyses showed the effects of the constraints limiting the number of truck entries and storage capacity, as well as convergence of the applied metaheuristic to approximately same local minima when these constraints are nonbinding.

The proposed model could be used without significant changes to optimize drayage operations for port terminals which include container transport only. The current model can be applied to container transport if we assume there are sufficient numbers of chassis at the intermodal terminal, or it may be extended to consider availability of this resource. Another interesting extension of this work would be to consider the correlation between truck roundtrips which would considerably increase the complexity of the proposed probabilistic analysis.
Finally, including reordering of truck roundtrips could further improve the solution if the operational practice allows it.

Appendix

The appendix provides the proof by mathematical induction for the expected primary dwell time $E[PDT_{k,j}^l]$ in chapter 4. The proof consists of three steps.

**Step 1.** Proof for $n^l = 2$
To compute the expected primary dwell time for the case including two randomly distributed connection times, we need to condition on five possible outcomes that depend on the realization of the three random variables (i.e. $Y_{k,j}, T^l_1$, and $T^l_2$).

$$E[PDT^l_{k,j} (n''=2)] = E[PDT^l_{k,j} (n''=2) \mid t_0'' < Y_{k,j} < t^l_1''; t^l_1'' < T^l_1 < t^l_2''; t^l_2'' < T^l_2] +$$

$$+ E[PDT^l_{k,j} (n''=2) \mid t^l_1'' < Y_{k,j} < t^l_1''; t^l_1'' < T^l_1 < t^l_2''; t^l_2'' < T^l_2] +$$

$$+ E[PDT^l_{k,j} (n''=2) \mid t^l_1'' < Y_{k,j} < t^l_1''; t^l_1'' < T^l_1 < y_{k,j}; y_{k,j} < T^l_1; t^l_2'' < T^l_2] +$$

$$+ E[PDT^l_{k,j} (n''=2) \mid t^l_1'' < Y_{k,j} < t^l_1''; t^l_1'' < T^l_1; y_{k,j} < T^l_1; t^l_2'' < T^l_2] +$$

$$+ E[PDT^l_{k,j} (n''=2) \mid t^l_1'' < Y_{k,j} < t^l_1''; t^l_1'' < T^l_1; y_{k,j} < T^l_1; t^l_2'' < T^l_2]$$

This expectation can be computed as follows in (A2).

$$E[PDT^l_{k,j} (n''=2)] = \int_{t^l_1''}^{t^l_2''} \int_{t^l_1''}^{t^l_2''} (t^l_1 - y_{k,j}) \cdot g_{k,j} (y_{k,j}) \cdot h^l_1 (t^l_1) \cdot h^l_2 (t^l_2) \cdot dt^l_2 \cdot dt^l_1 \cdot dy_{k,j} +$$

$$+ \int_{t^l_1''}^{t^l_2''} \int_{t^l_1''}^{t^l_2''} (t^l_2 - y_{k,j}) \cdot g_{k,j} (y_{k,j}) \cdot h^l_1 (t^l_1) \cdot h^l_2 (t^l_2) \cdot dt^l_2 \cdot dt^l_1 \cdot dy_{k,j} +$$

$$+ \int_{t^l_1''}^{t^l_2''} \int_{t^l_1''}^{t^l_2''} (t^l_1 - y_{k,j}) \cdot g_{k,j} (y_{k,j}) \cdot h^l_1 (t^l_1) \cdot h^l_2 (t^l_2) \cdot dt^l_2 \cdot dt^l_1 \cdot dy_{k,j} +$$

$$+ \int_{t^l_1''}^{t^l_2''} \int_{t^l_1''}^{t^l_2''} (t^l_2 - y_{k,j}) \cdot g_{k,j} (y_{k,j}) \cdot h^l_1 (t^l_1) \cdot h^l_2 (t^l_2) \cdot dt^l_2 \cdot dt^l_1 \cdot dy_{k,j} +$$

$$+ \int_{t^l_1''}^{t^l_2''} \int_{t^l_1''}^{t^l_2''} (t^l_1 - y_{k,j}) \cdot g_{k,j} (y_{k,j}) \cdot h^l_1 (t^l_1) \cdot h^l_2 (t^l_2) \cdot dt^l_2 \cdot dt^l_1 \cdot dy_{k,j} +$$

We can reduce the dimensions of four integrals from (A2) to obtain the following:
\[ E[PDT^l_{k,j} (n'=2)] = \int \int (t'_i - y_{k,j}) \cdot g_{k,j}(y_{k,j}) \cdot h'_i(t'_i) \cdot dt'_i \cdot dy_{k,j} + \]
\[ + \int \int (t'_i - y_{k,j}) \cdot g_{k,j}(y_{k,j}) \cdot h'_i(t'_i) \cdot dt'_i \cdot dy_{k,j} + \]
\[ + \int \int (t'_2 - y_{k,j}) \cdot g_{k,j}(y_{k,j}) \cdot h'_2(t'_2) \cdot dt'_2 \cdot dy_{k,j} + \]
\[ + \int \int (t'_2 - y_{k,j}) \cdot g_{k,j}(y_{k,j}) \cdot h'_2(t'_2) \cdot dt'_2 \cdot dy_{k,j} + \]
\[ + \int \int (t'_2 - y_{k,j}) \cdot g_{k,j}(y_{k,j}) \cdot h'_2(t'_2) \cdot dt'_2 \cdot dy_{k,j} + \]
\[ \] 
\[ E[PDT^l_{k,j} (n'=2)] = \sum_{i=1}^{2} \int \int (t'_i - y_{k,j}) \cdot g_{k,j}(y_{k,j}) \cdot h'_i(t'_i) \cdot dt'_i \cdot dy_{k,j} \]
\[ + \sum_{i=1}^{2} \int \int (t'_i - y_{k,j}) \cdot g_{k,j}(y_{k,j}) \cdot h'_i(t'_i) \cdot dt'_i \cdot dy_{k,j} \]
\[ + \sum_{i=1}^{2} \int \int (t'_i - y_{k,j}) \cdot g_{k,j}(y_{k,j}) \cdot h'_i(t'_i) \cdot h'_i(t'_i) \cdot dt'_i \cdot dt'_i \cdot dy_{k,j} \]
\[ \] 
\[ \text{Step 2. We make the inductive assumption that (A5) holds for } n' = n', \text{ which corresponds to equation 3.} \]
Step 3. Proof for $n^l = n + 1$

$$E[PDT_{k,j}^{l} (n^l = n+1)] = E[PDT_{k,j}^{l} (n^l = n)] +$$

$$+ E[PDT_{k,j}^{l} | t_n^l < Y_{k,j} < t_n^{l+1}, t_{n+1}^l < T_n^l < t_{n+1}^{l+1}, t_n^l < T_n^{l+1} < t_{n+1}^{l+1}], \quad (A6)$$

Using the inductive assumption stated in (A5), we compute expectation (A6) as follows:

$$E[PDT_{k,j}^{l} (n^l = n+1)] = \sum_{i=1}^{n} \int_{t_i^l}^{t_{i+1}^l} \int_{t_i^l}^{t_{i+1}^l} (t_i^l - y_{k,j}) \cdot g_{k,j}(y_{k,j}) \cdot h_i^l(t_i^l) \cdot dt_i^l \cdot dy_{k,j} +$$

$$+ \sum_{i=1}^{n} \int_{t_i^l}^{t_{i+1}^l} \int_{t_i^l}^{t_{i+1}^l} (t_i^l - y_{k,j}) \cdot g_{k,j}(y_{k,j}) \cdot h_i^l(t_i^l) \cdot dt_i^l \cdot dy_{k,j} +$$

$$+ \sum_{i=1}^{n} \int_{t_i^l}^{t_{i+1}^l} \int_{t_i^l}^{t_{i+1}^l} (t_i^l - y_{k,j}) \cdot g_{k,j}(y_{k,j}) \cdot h_i^l(t_i^l) \cdot h_{i+1}^l(t_{i+1}^l) \cdot dt_{i+1}^l \cdot dt_i^l \cdot dy_{k,j} +$$

$$+ \sum_{i=1}^{n} \int_{t_i^l}^{t_{i+1}^l} \int_{t_i^l}^{t_{i+1}^l} (t_i^l - y_{k,j}) \cdot g_{k,j}(y_{k,j}) \cdot h_i^l(t_i^l) \cdot h_{i+1}^l(t_{i+1}^l) \cdot dt_{i+1}^l \cdot dt_i^l \cdot dy_{k,j} +$$

$$+ \sum_{i=1}^{n} \int_{t_i^l}^{t_{i+1}^l} \int_{t_i^l}^{t_{i+1}^l} (t_i^l - y_{k,j}) \cdot g_{k,j}(y_{k,j}) \cdot h_i^l(t_i^l) \cdot h_{i+1}^l(t_{i+1}^l) \cdot dt_{i+1}^l \cdot dt_i^l \cdot dy_{k,j} +$$

$$+ \sum_{i=1}^{n} \int_{t_i^l}^{t_{i+1}^l} \int_{t_i^l}^{t_{i+1}^l} (t_i^l - y_{k,j}) \cdot g_{k,j}(y_{k,j}) \cdot h_i^l(t_i^l) \cdot h_{i+1}^l(t_{i+1}^l) \cdot dt_{i+1}^l \cdot dt_i^l \cdot dy_{k,j} +$$

The dimensions of the last two integrals in (A7) can be reduced as given in (A8).
implies that it holds for \( n' \). This completes the proof of Proposition 1. \( \square \)
References


• Cityporto. n.d. Cityporto Padova: Freight mobility in urban areas, A successful model of
citylogistics.

consolidation and last mile goods delivery by freight-tricycle in Manhattan" Opportunities and challenges. Transportation Research Board Annual Meeting. Washington, D.C.


EUROPLATFORMS EEIG. n.d. "Logistics centres: Directions for use."


  

  
     http://www.jada.org/.

- Jarzemskis, A., 2007. Research on public logistics centre as tool for cooperation, 
  


two-echelon inventory system with dependent supply uncertainty. *Transportation Research Part B* 45(8), 1128–1151.

• McKinnon, A. 1998. International review of urban transshipment studies and initiatives, UK.


http://www.bestufs.net.


